then the same moving charges also give a contribution to the magnetizing force **H** whose curl is given by the equation curl $\mathbf{H}=\partial D/\partial t=\epsilon_0(\partial \mathbf{E}/\partial t)+(\partial \mathbf{P}/\partial t)$. It is not necessary to state that it is the displacement "current" which gives rise to the magnetic field, it is sufficient to say that one effect is accompanied by the other. They have a *common cause*, namely, moving charges, at some earlier time. For a full discussion of this viewpoint the reader is referred to O'Rahilly.[4]

In the same way, if charges moving in the past give rise to a varying magnetic induction, then the same moving charges also give a contribution to the resultant electric field **E** whose curl is given by the equation

$$\text{curl}\mathbf{E}=-\partial \mathbf{B}/\partial t.$$

It is not the varying magnetic flux which gives rise to the induced electric field. The induced electric field and the varying magnetic field have a common cause in the past, namely, moving charges.

[1] A. P. French and J. R. Tessman, Am. J. Phys. **31**, 201 (1963).
[2] F. W. Sears, Am. J. Phys. **31**, 439 (1963).
[3] E. Whittaker, *A History of the Theories of Aether and Electricity* (Thomas Nelson and Son, New York, 1951), Vol. 1, p. 250.
[4] A. O'Rahilly, *Electromagnetics* (Longmans Green and Company, Ltd., London, 1938).

Rubber Band Experiment in Thermodynamics

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A SIMPLE laboratory experiment using the mathematical relationship of thermodynamics as derived from partial differential calculus can be made by third-year physics students. This experiment is performed on a rubber band and demonstrates, within experimental error, the validity of the thermodynamically imposed relations between the partial derivatives that involve the variables: temperature T, length L, and tension F.

A somewhat similar experiment was suggested by J. B. Brown[1] and Sutton,[2] where kg weights are hung on large rubber bands. The bands, heated by radiation from an infrared lamp, contract and lift the weights about a centimeter, providing a measure of $(\delta L/\delta T)_F$.

The experiment suggested in this report goes somewhat further but retains the basic simplicity of approach. It has been found to be particularly effective in demonstrating the physical reality associated with the abstract thermodynamic symbolism.

In a perfectly elastic system, all particles deformed from their equilibrium positions return to equilibrium once the stresses are removed. It is assumed that rubber has the property of perfect elasticity. The rational description of the experiment follows from the complete thermodynamic description of the rubber band in terms of F, L, and T. Assuming the equation of state is of the form

$$f(F,L,T), \tag{1}$$

one can derive that

$$(\delta F/\delta L)_T(\delta L/\delta T)_F(\delta T/\delta F)_L=-1. \tag{2}$$

The equipment necessary for this experiment is a large beaker, ruler, rubber bands, water, string, thermometer, heater, weights, and weight hanger.

The three partial derivatives of Eq. (2) may be evaluated in three independent measurements to verify this relationship experimentally. Each measurement must be made several times to reduce random errors. Several experimental problems arise in the measurements, which offer the student quite a challenge in experimental techniques.

This experiment was performed by third year students at the Texas A & M University with the following typical results from a 3-in. rubber band,

$$(\Delta F/\Delta L)_T\simeq 50.1 \text{ g/cm}$$
$$(\Delta L/\Delta T)_F\simeq -0.0023 \text{ cm/°C}$$
$$(\Delta T/\Delta F)_L\simeq 7.65 \text{ °C/g}$$

and

$$(\delta F/\delta L)_T(\delta L/\delta T)_F(\delta T/\delta F)_L\simeq -0.88.$$

The results are usually moderately successful regardless of the crude experimental apparatus one may be restricted to use, but still, the results represent a successful verification of the partial differential calculus as applied to the thermodynamics of a rubber band.

[1] J. B. Brown, Am. J. Phys. **31**, 397 (1963).
[2] R. M. Sutton, *Demonstration Experiments in Physics* (McGraw-Hill Book Company, Inc., New York, 1938).

Demonstration of Penetration of Potential Barriers

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CLASS demonstrations to accompany topics in quantum mechanics are rather scarce. But the topic of penetration or tunneling of a particle through a potential barrier can be effectively illustrated to a class through analogies with the behavior of water waves, light waves, and microwaves under certain conditions.

A good demonstration of the transmission of water waves across thin barriers is given in the movie film *Barrier Penetration of Waves*, which is produced and sold by Educational Services Incorporated.[1] The film shows that water ripples do not cross a wide barrier formed by a channel of deep water. But as the barrier narrows, the ripples are transmitted across it. This film is available in a Technicolor "Magi-cartridge" that forms a continuous loop that repeats every $2\frac{1}{2}$ min.

Another demonstration of barrier penetration makes use of frustrated total internal reflection of electromagnetic waves. It also illustrates the existence of what the British physicists call "evanescent waves" beyond the plane of total internal reflection. These are analogous to the Ψ waves inside a potential barrier whose height V exceeds the particle total energy E.

This demonstration may be shown with a 3-cm microwave transmitter, a receiver which feeds an audio amplifier or a vacuum tube voltmeter,[2] and two right-angle

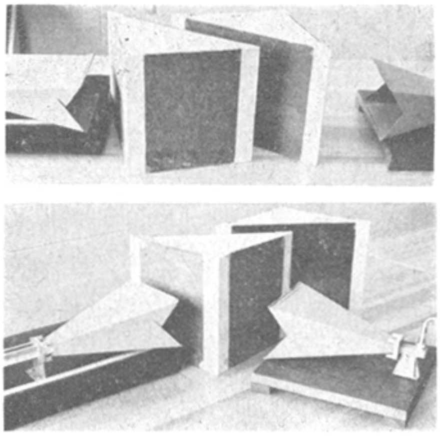

FIG. 1. Prisms and microwave horns.

paraffin prisms. Our prisms were made by molding paraffin in boxes with Masonite sides (Figs. 1 and 2). The microwave beam is sent through one of the small faces of the prism and made to strike the large face at an angle of about 55°, which assures total internal reflection. We show the class that no radiation can be detected at a distance of more than 10 cm beyond the single prism in line with the incident beam.

A second prism is brought up and placed with its large face parallel to the first to form a gap of about 10 cm (Fig. 2). Again, no transmission of radiation can be detected. The second prism is moved closer to the first. At a gap width of 5 cm, measured along the line of the incident beam, transmission is first detected. As the gap is narrowed the intensity of the transmitted beam increases remarkably.

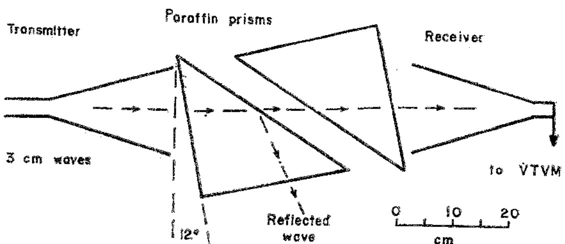

FIG. 2. Frustrated total internal reflection of microwaves.

As the gap is narrowed from 4 to 1 cm, there is a 20-dB gain of signal strength. To show that there is a corresponding loss in the reflected signal, the detector is moved around to pick up the reflected wave and the gap is again changed from 4 to 1 cm.

This demonstration is more than an illustration that is *per accidens* similar to quantum mechanical tunneling. There is a strict analogy between the Ψ wave for tunneling and the electromagnetic wave for frustrated total internal reflection. Eisberg has a good discussion of this point.[3] The time-independent Schrödinger equation is the same as the time-independent equation that governs the path of electromagnetic waves. The relation of the index of refraction μ to the usual quantum-mechanical parameters is given by

$$\mu(x) = (c/2\pi\nu)\{2m/h^2[E - V(x)]\}^{\frac{1}{2}}.$$

It is known from optical theory that the index of refraction of the region within the gap is an imaginary number in the case of total internal reflection. According to the above equation, this is equivalent to the condition that $E < V$, which corresponds to tunneling in the quantum mechanical case.

Sommerfeld points out that the above demonstration with microwaves was done at the Bose Institute in Calcutta before 1927 (perhaps as early as 1897) with 20-cm waves and asphalt prisms.[4] In an appendix to Strong's optics book, Hull describes a quantitative experiment on the evanescent microwaves that emerge from a totally reflecting surface.[5]

[1] 47 Galen Street, Watertown 72, Massachusetts.
[2] We use the Budd-Stanley model X4100 transmitter and receiver, Welch No. 2640.
[3] R. M. Eisberg, *Fundamentals of Modern Physics* (John Wiley & Sons, Inc., New York, 1961), p. 235.
[4] A. Sommerfeld, *Optics* (Academic Press Inc., New York, 1954), p. 32.
[5] J. Strong, *Concepts of Classical Optics* (W. H. Freeman and Company, San Francisco, 1958), p. 517.

AMERICAN JOURNAL OF PHYSICS VOLUME 31, NUMBER 10 OCTOBER 1963

ANNOUNCEMENTS AND NEWS

Book Reviews

Josiah Willard Gibbs: The History of a Great Mind. LYNDE PHELPS WHEELER. Pp. 270, Yale University, New Haven, 1962. Price $1.75 (paperback).

Just one hundred years ago Josiah Willard Gibbs received the second Ph.D. degree in science given in the U. S., but what most of us physicists probably do not recall is that he received it in engineering. Comparatively few physicists, I suppose, realize that Gibbs' thesis was concerned with a geometrical approach to the art of gear design, that his first two ventures in research were in applied mechanics, directed toward invention, and that he actually received a patent for an invention of "an improved railway car brake." Such interesting details are brought out in the biography by the author, who was a student of Gibbs in the nineties. His primary objective is to explore "the history of a great mind," the subtitle of