# Frustrated total internal reflection: A demonstration and review

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Frustrated total internal reflection (FTIR) has been studied since the time of Newton and Fresnel. We review its history and applications in modern optics. A simple theoretical description of the phenomenon is presented using Maxwell's equations. The analogy (often made in textbooks) between FTIR and quantum mechanical tunneling in one dimension is discussed. A simple experimental apparatus, suitable for a laboratory demonstration, is described and a quantitative comparison of the theory with experiment is made at optical wavelengths. A He–Ne laser, power meter, and a simply constructed double-prism arrangement are used for the demonstration.

## I. HISTORICAL INTRODUCTION

When light is incident on the interface of two media, it is partly transmitted into the second medium and partly reflected back into the first. If, however, the index of refraction of medium 1 ($n_1$) is greater than the index of medium 2 ($n_2$) and the angle of incidence exceeds the critical angle [$\phi_c = \sin^{-1}(n_2/n_1)$], total internal reflection occurs. The incident light is reflected completely back into medium 1. The Fresnel relations[1] characterize the transmitted and reflected waves (for linear media) in terms of the incident wave amplitude, phase, and polarization.

An intriguing phenomenon is the penetration of the wave (evanescent wave) or "disturbance" into the second medium when total reflection occurs. Newton[2] found the phenomenon fascinating, as did Fresnel, Verdet, Young, Huygens, Biot, Stokes, and Quincke. References to their works are contained in the comprehensive paper by Hall[3] (*not* of the Hall effect), who reported in 1902 on thorough experimental and theoretical investigations of the phenomenon. The theoretical exposition relied heavily on the slightly earlier (1900) presentation by Drude.[4] To investigate the effect experimentally, Hall chose the most common solution: the introduction of a third medium (index $n_3$) close to the first one (Fig. 1). The second medium is now a thin film with a thickness of the order of the wavelength of the light employed. The total reflection of light is thus "frustrated," a term coined by Leurgans and Turner[5] in 1947. All three media are assumed transparent at the wavelength of operation. If the second medium is absorbing, the term attenuated total reflection (ATR) has been used.

Hall examined the dependence of the penetration depth on the angle of incidence and polarization of the incident radiation for a number of different media. He also presented a theoretical calculation for the transmission coefficient of the incident light. With regard to the experimental verification of the relation between the transmission coefficient and the separation of media 1 and 3 Hall writes: "There seems at present no method for experimentally testing the theory for two media in the case of light waves. It would seem feasible, however, to test the theory with short electric waves. This the writer hopes to do at some future time."

Hall was perhaps unaware that exactly such experiments had been performed by Bose as early as 1897. In an ingenious experiment with two right angle prisms and one of the earliest generators of centimeter wave electromagnetic radiation, Bose examined the penetration of the waves by using the now canonical double-prism arrangement. His paper was communicated by Lord Rayleigh, and appeared in the *Proceedings of the Royal Society*.[6]

Bose's pioneering experiment was repeated with centimeter wave radiation by Schaefer and Gross,[7] Brady *et al.*,[8] and Culshaw and Jones,[9] who quantitatively verified the transmission coefficient versus separation predictions of the theory based on the Maxwell equations.

Wood[10] and Raman[11] devoted some of their time to FTIR. Wood used FTIR techniques "to establish the granular nature of certain metallic films."

Advances in the theoretical understanding beyond the work of Hall were made by Eichenwald,[12] Foersterling,[13] and Arzelies,[14] who studied the energy flow and arrived at the notion of the exponentially decaying evanescent wave. The effect of a finite cross section for the incident beam, rather than an infinite plane wave, was discussed by Picht.[15]

Despite these theoretical advances, the discovery of the Goos–Hanchen shift in 1947[16] caused some controversy. The Goos–Hanchen shift is the displacement $d_0$ of the totally reflected wave, as shown in Fig. 2. A discussion of the controversy, and a clear theoretical treatment, has been given by Renard.[17] The Goos–Hanchen shift has been found important in the theory of waveguides.[18]

The discovery of barrier penetration in quantum mechanics brought the insight that it was a phenomenon rather analogous to FTIR. Hence the name "optical tunneling" for FTIR which is used fairly often in quantum mechanics texts.[19] It has, in fact, become quite popular to quote FTIR

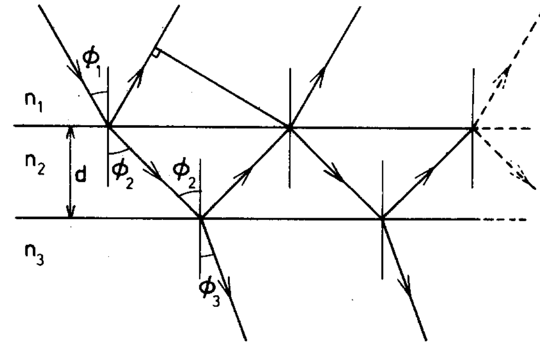


Fig. 1. Reflection and transmission at a thin, homogeneous film.

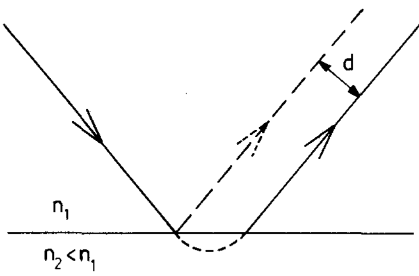


Fig. 2. The Goos–Hanchen shift.

as an illustration of quantum mechanical tunneling. We comment on this analogy in Sec. II.

FTIR has proved to be of great practical use in recent years. An entire branch of spectroscopy has evolved around it, reviewed in depth by Harrick.[20] The study of thin films, both metal and dielectric, has been advanced through the application of FTIR and ATR techniques. The discovery of the Otto[21] and Kretschmann[22] configurations for internal reflection spectroscopy using lasers have been of great importance in the study of surface waves.[23] The recent elegant experiments of Shen, DeMartini and co-workers, and the experiments and theories of a host of other groups in the area of the nonlinear generation of surface waves has led to the opening of a new subject: surface nonlinear optics.[24] The pioneering papers of Bloembergen and co-workers[25] led to the first theoretical understanding of total reflection phenomena at the surface of nonlinear media. The suggestion by Kaplan[26] of bistable devices based on reflection from nonlinear interfaces has also stimulated a great deal of theoretical and experimental activity.[27]

One of the applications of FTIR has been the design of laser resonators. Court and von Willisen[28] have presented an admirable review of this subject. Subsequently, further developments were made by Aleksoff and Harrick,[29] Steele *et al.*,[30] Polloni,[31] Marowsky,[32] and Wilner and Murarka[33] in the area of $Q$ switches, light modulators, and dye laser systems with variable output coupling. Our own interest in FTIR was motivated by the problem of developing a versatile variable transmission output coupler for ring laser systems. Another application is to the design of optical filters.[34]

The many experiments mentioned previously demonstrate FTIR at both optical and microwave frequencies.[2,3,6–11] The microwave experiments[6–11] are quantitative measurements of the transmission coefficient versus separation relation, but lack the visual impact achieved in the optical (visible) wavelength regime. The experiments at visible wavelengths do not usually quantitatively verify the results of the theory. An exception was the photon-counting experiment performed by Coon in 1966.[35] Coon's experiment showed the agreement of theory and experiment when the separation of the prism surfaces (he used the canonical double-right angle prism arrangement) was between 3.5 and 8.0 wavelengths of the 546.1-nm-Hg line used. His instrumentation was quite sophisticated for most undergraduate laboratories in that he used a cooled photomultiplier, pulse height analyzer, and a grating monochromator.

A series of beautiful experiments to measure the phase shifts undergone by the evanescent waves was done by Carniglia and Mandel in 1971.[36–38] Since the evanescent waves propagate in a direction parallel to the glass–air interface, there is no phase shift in the transmitted wave, as was experimentally verified. A quantized formalism for evanescent waves was also developed by these authors.[36]

A review of many applications of FTIR and evanescent waves in optical imaging has been published by Bryngdahl.[39]

In the present article, we report a technique for the demonstration of FTIR with a He–Ne laser, a powermeter or suitable linear response photodiode, and a simply constructed double-prism arrangement. Fairly reasonable quantitative measurements of the transmission due to FTIR are easily made with the apparatus, and the theoretical predictions may be verified over a wide range of separation of the prisms.

The prism arrangement traditionally employed in FTIR experiments usually consists of two right angle prisms with their hypoteneuses facing each other, separated by a small gap. We have departed from this tradition, and used two Pellin–Broca prisms for this purpose. The device thus constructed has recently been used as a variable transmission output coupler and tuner in a ring dye laser system and provides several advantages over traditional output coupling and tuning methods.[40] The coupler is free from extraneous losses, since the beam enters and exits the coupler at Brewster's angle. It can be used over a wide range of wavelengths, limited only by the material of the Pellin–Broca prisms, and with both cw or pulsed lasers. No coatings are used, hence high power operation is less likely to cause damage than with coated optics.

In Sec. II we present a short derivation of the transmission coefficient in a FTIR device. The results have been given by several authors[34,28]; we present the derivation (for clarity and completeness) starting with the Fresnel relations,[1] and compare our results with those obtained from the quantum barrier penetration problem. Our treatment follows that of Court and von Willisen[28] very closely.

In Sec. III we describe the apparatus which was used. Details of the construction are given. Sec. IV describes the measurements made and results obtained.

## II. THEORETICAL CONSIDERATIONS

The propagation of a plane electromagnetic wave through a thin homogeneous film which is surrounded by two semi-infinite dielectric media (possibly of different refractive indices) can be described through the use of the Fresnel relations.[1] These are a direct consequence of the Maxwell equations and the appropriate boundary conditions. The Fresnel relations connect the incident, reflected, and transmitted amplitudes of the fields in both sides of the gap, for polarizations parallel with and perpendicular to the plane of incidence of the light. These relations, for the first two media, are:

$$E_{\parallel}^{(t)} = \frac{2n_1 \cos \phi_1}{n_2 \cos \phi_1 + n_1 \cos \phi_2} E_{\parallel}^{(i)}, \tag{1}$$

$$E_{\parallel}^{(r)} = \frac{n_2 \cos \phi_1 - n_1 \cos \phi_2}{n_2 \cos \phi_1 + n_1 \cos \phi_2} E_{\parallel}^{(i)}, \tag{2}$$

$$E_{\perp}^{(t)} = \frac{2n_1 \cos \phi_1}{n_1 \cos \phi_1 + n_2 \cos \phi_2} E_{\perp}^{(i)}, \tag{3}$$

$$E_{\perp}^{(r)} = \frac{n_1 \cos \phi_1 - n_2 \cos \phi_2}{n_1 \cos \phi_1 + n_2 \cos \phi_2} E_{\perp}^{(i)}. \tag{4}$$

$E_{\parallel}^{(i)}$ and $E_{\perp}^{(i)}$ are the parallel and perpendicular components of the incident field, $\phi_1$ is the angle of incidence and $\phi_2$ is the transmission angle, and $n_1$ and $n_2$ are the refractive indices of the media on either side of the (1,2) interface. A similar set of relations applies to the fields on either side of the (1,3) interface. Figure 1 shows the physical situation. The phase difference between consecutive reflected beams is easily calculated. The path difference is $\Delta l = 2n_2 d \cos\phi_2$, and hence the phase change is $\delta = k\Delta l$. Here $k = 2\pi/\lambda$ is the propagation number. Using Snell's law, $\delta = (4\pi d/\lambda)\,(n_2^2 - n_1^2 \sin^2\phi_1)^{1/2}$.

The Stokes' relations[1] connect the transmission and reflection coefficients $t_{12}$, $r_{12}$ for light originating in the $n_1$ medium and the coefficients $t_{21}$, $r_{21}$ for light originating in the $n_2$ medium. These are

$$t_{12}t_{21} + r_{12}^2 = 1\,, \tag{5}$$

$$r_{12} = -r_{21}\,. \tag{6}$$

The reflected wave amplitudes [from the (1,2) interface] are easily calculated from the incident amplitude $E^{(i)}$. They are $r_{12}E^{(i)}$, $t_{12}e^{i\delta}$, $t_{21}r_{23}E^{(i)}$, $t_{21}r_{21}r_{23}^2 t_{12}E^{(i)} e^{2i\delta}$ ..., where we have defined $r_{23}$ as the amplitude reflection coefficient for light incident on the (2,3) interface from medium 2. We add these multiple reflection contributions and sum the geometric series, while making use of the Stokes' relations to get the total reflected amplitude $E^{(r)}$:

$$E^{(r)} = E^{(i)}[(r_{12} + r_{23}e^{i\delta})/(1 + r_{12}r_{23}e^{i\delta})]\,. \tag{7}$$

Define

$$re^{i\theta} = (E^{(r)}/E^{(i)}),\quad \text{i.e.,}$$

$$re^{i\theta} = (r_{12} + r_{23}e^{i\delta})/(1 + r_{12}r_{23}e^{i\delta})\,. \tag{8}$$

We may now treat specifically the case of FTIR. Then, $\delta = (4\pi d/\lambda).\ (n_2^2 - n_1^2\sin^2\phi_1)^{1/2}$ becomes imaginary, since $n_1 \sin\phi_1 \geqslant n_2$, i.e., light is incident at greater than the critical angle. Let us rewrite

$$\delta = i\delta'\,, \tag{9}$$

where

$$\delta' = (4\pi d/\lambda)(n_1^2\sin^2\phi_1 - n_2^2)^{1/2} \tag{10}$$

is real.

Also,

$$r_{12} = e^{i\delta_{12}} \tag{11}$$

and

$$r_{23} = -r_{32} = -e^{i\delta_{32}}\,. \tag{12}$$

From these two relations, we may re-express $re^{i\theta}$ as follows:

$$re^{i\theta} = (e^{i\delta_{12}} - e^{i\delta_{32}} - e^{-\delta'})/(1 - e^{i\delta_{12}}e^{i\delta_{32}}e^{-\delta'})\,. \tag{13}$$

Now,

$$(r_{12})_{\parallel} = (E_{\parallel}^{(r)}/E_{\parallel}^{(i)})_{12} = \frac{n_2\cos\phi_1 - n_1\cos\phi_2}{n_2\cos\phi_1 + n_1\cos\phi_2} = e^{i\delta_{12}} \tag{14}$$

and

$$(r_{32})_{\parallel} = (E_{\parallel}^{(r)}/E_{\parallel}^{(i)})_{32} = \frac{n_2\cos\phi_3 - n_3\cos\phi_2}{n_2\cos\phi_3 + n_3\cos\phi_2} = e^{i\delta_{32}} \tag{15}$$

for the parallel component of the incident and reflected fields. Similarly, one can write down the expressions for $(r_{12})_{\perp}$ and $(r_{32})_{\perp}$.

If one substitutes these expressions into the equation for $re^{i\theta}$ we may calculate the transmission coefficient $T = 1 - |r|^2$ for both the parallel and perpendicular components of the field. One then obtains[28]

$$1/T = \alpha\sinh^2 y + \beta\,, \tag{16}$$

where

$$y = (\delta'/2) = (2\pi d/\lambda)(n_1^2\sin^2\phi_1 - n_2^2)^{1/2} \tag{17}$$

and $\alpha$ and $\beta$ take the forms $\alpha_{\perp}$, $\alpha_{\parallel}$, $\beta_{\perp}$, and $\beta_{\parallel}$:

$$\alpha_{\perp} = \frac{(N^2-1)(n^2N^2-1)}{4N^2\cos\phi_1(N^2\sin^2\phi_1 - 1)(n^2 - \sin^2\phi_1)^{1/2}}\,, \tag{18}$$

$$\beta_{\perp} = \frac{[(n^2 - \sin^2\phi_1)^{1/2} + \cos\phi_1]^2}{4\cos\phi_1(n^2 - \sin^2\phi_1)^{1/2}}\,, \tag{19}$$

$$\alpha_{\parallel} = (\alpha_{\perp}/n^2)[(N^2+1)\sin^2\phi_1 - 1] \times [(n^2N^2+1)\sin^2\phi_1 - n^2]\,, \tag{20}$$

$$\beta_{\parallel} = \frac{[(n^2 - \sin^2\phi_1)^{1/2} + n^2\cos\phi_1]^2}{4n^2\cos\phi_1(n^2 - \sin^2\phi_1)^{1/2}}\,. \tag{21}$$

Here,

$$n \equiv (n_3/n_1) \quad\text{and}\quad N \equiv (n_1/n_2)\,. \tag{22}$$

In Fig. 3(a) we have plotted $T_{\parallel}$ vs $(d/\lambda)$ for the incident $E$ field parallel to the plane of incidence for an angle of incidence of 45°. Two different curves for $n_1 = n_3 = 1.65$ and $n_1 = n_3 = 1.517$ are shown, the latter since 1.517 is the refractive index of the borosilicate crown glass of which our Pellin–Broca prisms are constructed. For $(d/\lambda)$ greater than unity, the transmission coefficient falls off rapidly in an exponential fashion. This was the regime tested by Coon in his photon-counting experiments on FTIR; his measurements were confined to $3.5 \leqslant (d/\lambda) \leqslant 8.0$.

The curves of $T_{\perp}$ vs $(d/\lambda)$ are similar in nature to those shown in Fig. 3(b), although there are quantitative differences.

We now briefly comment on the similarity of quantum mechanical barrier penetration to FTIR. If the special case $n_0 \equiv n_1 = n_3$ and $n_2 = 1$ is considered, we obtain the simplified expressions

$$T = 1/(\alpha\sinh^2 y + 1)\,, \tag{23}$$

$$\alpha_{\perp} = [(n_0^2 - 1)/2n_0]^2\{1/[\cos^2\phi_1(n_0^2\sin^2\phi_1 - 1)]\}\,, \tag{24}$$

$$\alpha_{\parallel} = \alpha_{\perp}[(n_0^2 + 1)\sin^2\phi_1 - 1]^2\,, \tag{25}$$

with

$$\beta_{\perp} = \beta_{\parallel} = 1 \tag{26}$$

and

$$y = (2\pi d/\lambda)(n_0^2\sin^2\phi_1 - 1)^{1/2}\,. \tag{27}$$

We compare this expression with that for the transmission coefficient for a particle incident on a square potential barrier of height $V_0$ and width $d$, treated in standard quantum mechanics texts (see, for example, Schiff, Ref. 41);

$$T = 1/\{1 + V_0^2\sinh^2\gamma d/[4E(V_0 - E)]\}\,, \tag{28}$$

where

$$\gamma = (2m(V_0 - E)/\hbar^2)^{1/2}\,. \tag{29}$$

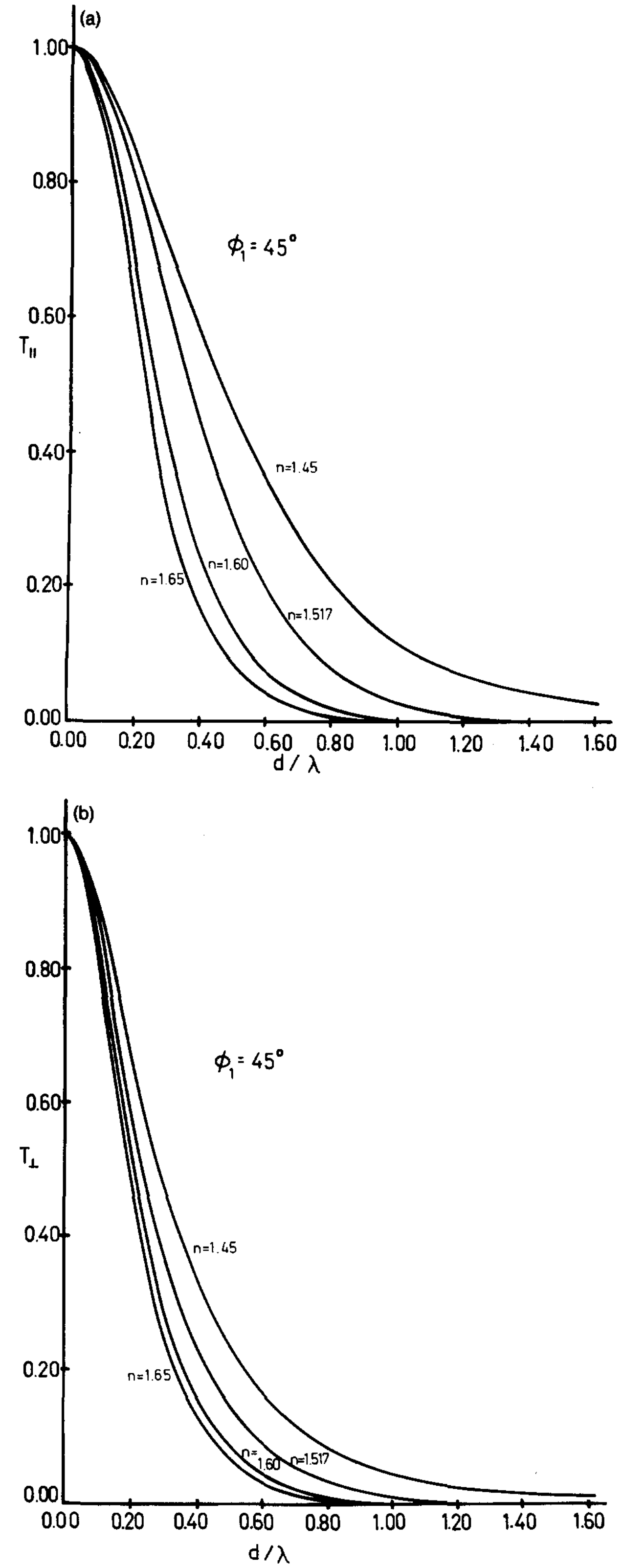

Fig. 3. (a) $T_{\parallel}$ vs $(d/\lambda)$ for different refractive indices; (b) $T_{\perp}$ vs $(d/\lambda)$.

If we take the correspondence[42]:

$$(mE/h^2) \rightarrow (n_0^2/4\lambda^2)\,, \tag{30}$$

$$(mV_0/h^2) \rightarrow (n_0^2 - 1)/(2\lambda^2)\,, \tag{31}$$

we find that Eq. (28) is identical to Eq. (23) with $\phi_1 = 45°$, for polarization *perpendicular* to the plane of incidence. The correspondence is not exact for the parallel polarization. We emphasize the different dimensionalities of the two problems. Equations (28) and (29) pertain to a one-dimensional barrier penetration situation, whereas the FTIR is richer not only in spatial dimensions, but contains the polarization of the light waves as an additional factor which has to be considered. Eisberg and Resnick[19] have noted that the *one-dimensional* wave equation for propagation in a medium with a position dependent refractive index $n(x)$,

$$\left(\frac{d^2\psi}{dx^2}\right) + \left(\frac{2\pi n(x)}{\lambda}\right)^2 \psi(x) = 0\,, \tag{32}$$

where

$$n(x) \rightarrow (\lambda/2\pi)\{2m[E - V(x)]/\hbar^2\}^{1/2} \tag{33}$$

is identical in form to the time-independent one-dimensional Schrödinger equation. We note that this is a mathematical analogy. The physical situations, described by the Maxwell equations and the Schrödinger equation, coincide in their mathematical description only for rather special circumstances.

Our entire treatment was based on monochromatic, infinite plane waves. Considerations which include the finite beam size effects are discussed in terms of "lateral waves" by Tamir,[43] who also explains the origin of the Goos–Hanchen shift in terms of the lateral wave. In our demonstration, this shift is not significant.

## III. THE APPARATUS

The double-prism device we have used is shown in Fig. 4. It consists of two Pellin–Broca prisms,[44] rather than the traditionally used right angle prisms, compressed against each other by a spring loaded mechanism. These are mounted on a magnetic base for easy placement on a magnetic top optical table. Any other convenient mounting scheme would be appropriate.

The advantage of using the Pellin–Broca prisms lies in the fact that all beams enter and exit at Brewster's angle, if so desired. Thus for polarization in the plane of incidence, there are no extraneous losses by reflection to contend with in the transmission coefficient measurements. This is an advantage over the use of right angle prisms where the beams are incident normally on the external prism faces. In our case, no antireflection coatings are needed. One may thus test the T vs $(d/\lambda)$ relation for the parallel component of the $E$ field in a remarkably "clean" measurement. The prisms are commercially available and the entire device can be quickly fabricated.

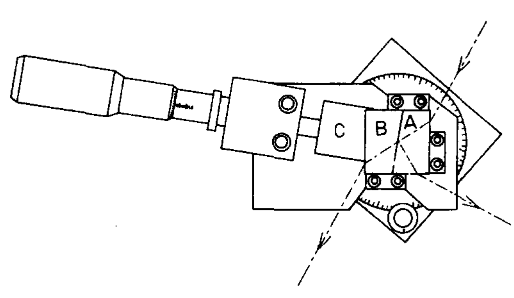

Fig. 4. Arrangement of the double-prism device.

Three short pieces of metal are bolted to the base plate so as to hold the prisms A and B in place. A micrometer screw exerts pressure on a spring contained with the block C. Screwing in the micrometer increases the pressure on prism B, and decreases the spacing between the two prisms. This increases the output coupling transmission coefficient. The spring inside block C gives us the ability to obtain small changes in the gap thickness $d$ with fairly large translations of the micrometer screw. The apparatus is constructed so that the compression is normal to the interface, with no shearing stress.

A Spectra-Physics model 159 laser with an output of 5 mW was used in our experiments. A polarizer was used to polarize the laser output parallel to the plane of incidence, reducing the available power by about a factor of 2. The Spectra-Physics 404 power meter is adequately sensitive for the transmission measurements. A suitable linear response photodiode is actually all that is needed if the sensitive area is large enough to receive the entire beam. A lens may be used to focus the beam if the active area is small. Relative power measurements only are of importance since the transmission coefficient (ratio of transmitted to incident intensities) is being measured.

The surfaces of the prisms must be thoroughly cleaned. If a grain of dust is caught between them, the experiment will not work. We have used lens tissue and pure (spectranalyzed) methanol to clean the surfaces before a measurement. The apparatus construction must be very rigid. One must also be careful to avoid backlash in the measurements of the micrometer position.

## IV. MEASUREMENTS AND RESULTS

The laser beam is polarized parallel to the plane of incidence and directed at the double-prism device so that the beam enters at Brewster's angle. This is ensured by adjusting the polarizer and the incidence angle until the reflection off the external prism surface is minimized (almost completely absent). The beam enters prism A, reflects at the interface, and a small part of it "tunnels" through to prism B and exits to be incident on the power meter (or photodiode).

The transmission coefficient $T$ is easily calculated from the measurements of the incident and transmitted laser intensities, as a function of the micrometer position. A graph of $T$ versus the micrometer position is plotted (Fig. 5). Such a graph is quite reproducible, although one must be careful to avoid backlash errors in reading the micrometer. A good heavy-duty micrometer screw is needed for the experiment to be repeatable with good accuracy.

Once the graph of $T$ versus micrometer position has been obtained, it is necessary to convert the micrometer reading to a corresponding gap separating the two prisms. To determine the gap, we used a simple interferometric measurement. The He–Ne laser was aimed at the same spot on the interface at which the transmission measurements were made, but this time almost at normal incidence. Part of the beam is reflected back from the surfaces of both prisms and interference fringes are formed which depend in brightness on the gap between the two prisms.

Starting with the minimum separation, we count dark fringes in the reflected beam (from the interface) and note the corresponding micrometer positions. The first minimum occurs at $d = \lambda/4$, and the other follows at $3\lambda/4$, $5\lambda/4$, $7\lambda/4$, $9\lambda/4$, etc. The results of such a measurement are

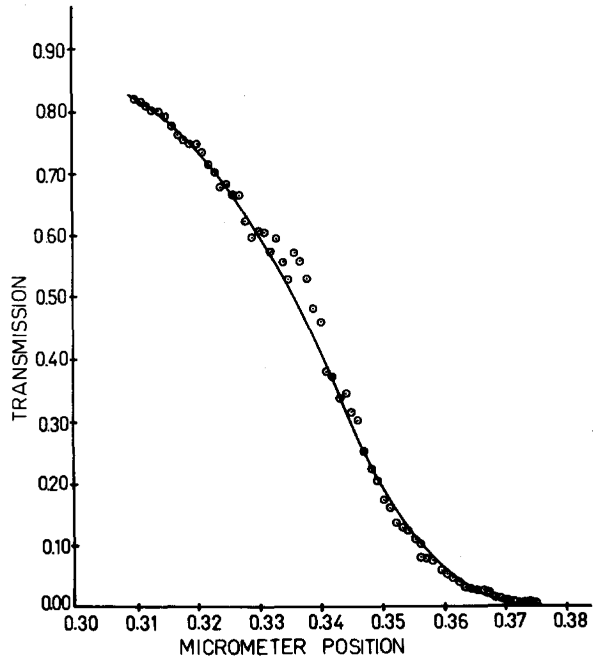


Fig. 5. Transmission coefficient $T$ versus micrometer position. The solid line is an empirical fit.

shown in Fig. 6. It is now easy to calculate the spacing of the prism surfaces for any given micrometer position by interpolation from this graph. If one wishes to be somewhat sophisticated, a cubic spline interpolation method[45] could be used, although this is not really necessary. A large number of repeated measurements result in a very smooth curve, as is shown in Fig. 6, where the average micrometer positions for three different measurements are shown.

We may now obtain the gap between the prism which corresponds to a given transmission coefficient. A plot of the coefficient $T$ vs $(d/\lambda)$ is shown in Fig. 7. The values of $(d/\lambda)$ range from 0.25–1.0, approximately. The corresponding values of $T$ range from about 80%–0.008%. The agreement between theory and experiment is not perfect,

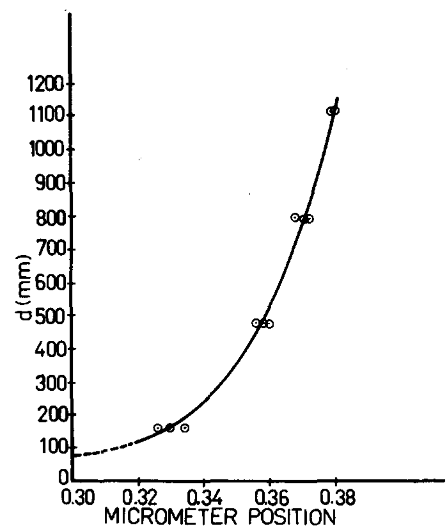


Fig. 6. Separation of gap [$d = (2l - 1)\lambda/4$; $l = 1,2,3,...$; $l$ is the order of minimum] versus micrometer position. The solid line is an empirical fit.

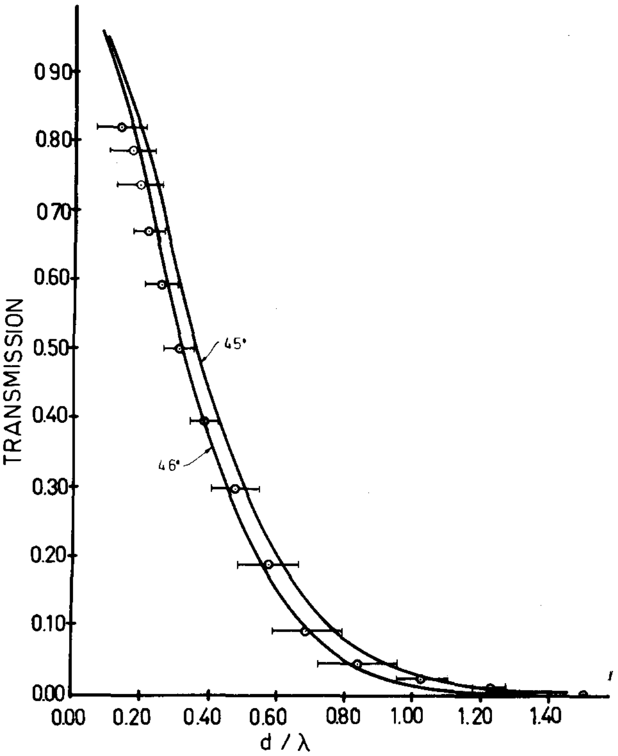


Fig. 7. Comparison of theoretical and experimental results of transmission versus $(d/\lambda)$ with incident angles 45° and 46°. The horizontal error bars are estimated errors in the determination of $(d/\lambda)$ from Figs. 5 and 6.

but reasonably good. To our knowledge such a measurement has not been done in the optical (visible) range of wavelengths.

We have also carried out measurements at incidence angles other than 45°. The transmission coefficient is quite sensitively dependent on the angle of incidence. Contaminant films are very difficult to avoid and could cause appreciable problems through scattering and absorption. Thus the glass surfaces need to be thoroughly cleaned.[3]

Similar experiments may be done with light polarized perpendicular to the plane of incidence. In that case corrections for the loss due to reflections at the external surfaces have to be made, and the measurements lose some of their simplicity. The same is true of other polarization states of the light. Different materials may be used for the two prisms, e.g., one of crown glass, the other flint. The essence of the physics is, however, best demonstrated by the simplest configuration.

This experiment serves as a striking visual demonstration of FTIR. The double Pellin–Broca prism arrangement has recently been used in our laboratory as an output coupler for a ring dye laser. In this situation the device acts as a variable transmission output coupler, as well as a tuner for the laser. The output wavelength of the laser may be tuned without any deviation of the output beam. The results of detailed measurements which characterize the behavior of the laser with such a coupler are being published elsewhere.[40]

## V. CONCLUSION

We hope to have convinced our readers that a simple experiment may be done to verify the phenomenon of frustrated total internal reflection. This phenomenon, although known for hundreds of years, has been usually demonstrated in a qualitative way at visible wavelengths. We hope to have clarified the analogy of FTIR to quantum mechanical barrier penetration. Finally, it should be pointed out that FTIR and ATR (attenuated total reflection) have important applications in spectroscopy, nonlinear optics, and integrated optics and may prove to be of much importance in the near future.

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# Fluid dynamic and kinetic theory models for a nonprovocative land defense of central Europe

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A major problem in using theory to make predictions in a practical situation is the validity of the theory. The theoretical predictions are only as good as the theory. A model is developed, based upon a one-dimensional equation of continuity, for predicting the effectiveness of an attritional mode of defense against a conventional land attack in Europe. The validity of the model and its predictions is tested by comparing the results with those of a stochastic, discrete model of multiple small defensive battles, with the latter results obtained by Monte Carlo type computer simulations. The concordance of the results supports the validity of the models and the efficacy of applying physical-type reasoning to pressing, nonphysical, world problems.

## I. INTRODUCTION

Many of today's physics students will not end up as professional physicists. Thus it should be useful for them to see the concepts, methods, and techniques of physics applied to other problems—specifically, to problems relevant to major concerns of the contemporary world such as military security. The relevance should help them focus their minds upon the problems and the physical methods of solution, as well as entice them into the physics courses. The application of the approach of theoretical physics to nonphysical problems has a long, well-documented history, e.g., operations research developed out of the World War II activities of some physicists.[1] In this paper I present another problem and solution which should be easy to present to students, should interest them on its own merits, and which raises the philosophical question as to how a theory is to be validated without an actual experimental test. The problem, to which theory is applied in this paper, is one of creating a nonconventional, nonprovocative, land defense of central Europe.

Makers of public policy, whether domestic or international, are inherently—if implicitly—engaged in making predictions: "...if I adopt this policy course, then I expect... ." Usually their predictions are just based upon the successes or failures of past policies—they are merely extrapolations from past experiences. But if the world in which the policy (and predictions) is to be made is radically different from the past world (as is likely in the international sphere since the advent of the possibility of the massive, prompt, long-range delivery of nuclear weapons[2]), simple extrapolations from the past are likely to be inadequate and perhaps dangerous.[3] In this case resort must be had to an alternative basis for prediction—theoretical models. As people who are professionally identified with the making of predictions, physicists should be particularly valuable in helping governmental policy makers and citizens deal with the contemporary world. (After all, physics might be said to stem from the success of Galileo's projectile predictions and Newton's planetary predictions.) As a profession which has had much to do with the making of modern weapons technology,[4] so much a shaper of and threat to the modern world, it seems appropriate that physicists—and their skills—be applied to helping shape, interpret, and justify contemporary international security policies.

We are accustomed to making predictions with confidence, based upon validated theories. For example, people did not venture forth into space without relying on gravitational and mechanical laws which had had three centuries of validation—validation being the successful agreement between previous predictions and experiment. Validation of theory is required for confident prediction, but successful prediction is required for validation. What is to be done if the validating experiments cannot be done without disasterous consequences? The falling apple which presumably validated Newton's theory of gravitation did not do much harm. A theory for a novel defense system for Europe which would require a European war to validate it is not going to be very useful for making predictions in today's

  