Measurement of the longitudinal shift of radiation at total internal reflection by microwave techniques*

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We have measured the longitudinal shift of a microwave beam at total internal reflection. Good agreement with the prediction of classical electromagnetic theory is provided. The magnitude of the effect at microwave frequencies is conveniently large and the experimental setup used is sufficiently straightforward to allow this experiment to be repeated in an undergraduate laboratory program.

INTRODUCTION

That a ray of light propagating in a region of higher index upon encountering total internal reflection will penetrate into the region of lower index and thus be shifted longitudinally has been known, or at least believed, since the time of Newton.[1] The first experimental work on this effect concerned the fact that total internal reflection can be "frustrated" by the presence of another layer of the higher index spaced a wavelength or so from the original interface. The first direct measurement of the longitudinal shift itself was by Goos and Hänchen in 1943,[2] and recent writers have named this effect in their honor. An extensive review article by Lotsch[3] lists 112 (multiple) references to work on the Goos–Hänchen effect, a testimonial to the continued interest of scientists from the time of Newton in this optical phenomenon. Interestingly, in the 30 years since the original Goos–Hänchen experiment, it has been repeated (and improved) only once.[4] Both of these experimental efforts have been with radiation in the visible region, have used multiple reflection techniques, and have been restricted to angles within about 1° of the critical angle. Since the effect is of the order of one wavelength, we were motivated by considerations of simplicity to measure the effect with microwaves.[5] Furthermore, we were interested in showing that the effect occurs on a single reflection—and for somewhat larger angles than had been previously used. With relatively simple apparatus we have been able to demonstrate that the effect is seen, and that its magnitude agrees with the classical theory.

THEORETICAL DISCUSSION

We shall give only a brief statement of the theoretical situation since there are numerous other references to various derivations of the formula for the longitudinal shift.[6] First let us summarize some salient points about the longitudinal shift.

(1) A plane electromagnetic wave of unlimited extent perpendicular to the propagation direction represents a rigorous solution of Maxwell's equations. Thus, matching boundary conditions at an interface, which implies that Snell's law is exactly obeyed, yields the Fresnel formula for reflection and transmission. But a plane wave has no longitudinal shift—simply because it has no reference point to shift.

(2) A wave that has limited spatial extent (a beam of light for example) perpendicular to the direction of propagation is an approximate solution to Maxwell's equations since it can be parameterized in terms of a series of plane wave solutions whose terms are proportional to the derivatives of the amplitude of the **E** or **H** vectors with respect to the appropriate perpendicular direction. Such a wave will suffer a longitudinal shift at total internal reflection.

(3) In the simplest exposition of the approximate development of the expression for the longitudinal shift of a beam whose transverse amplitude is slowly changing—the only surviving terms in the expansion of the incident beam are the unlimited plane wave term and the term proportional to the first derivative of the amplitude with respect to transverse position. In this approximation the expression for the shift is independent of the exact beam shape. Formulas for the longitudinal shift have been developed by a number of authors under these assumptions.[3] These formulas are known to be incorrect exactly at the critical angle, and also at grazing incidence.

(4) By including more terms in the expansion of the incident wave, one can get increasingly more precise values for the shift, but in practice the formulas derived under the assumptions in (3) are valid for all but the most exacting work.

(5) The longitudinal shift is a function of the polarization state of the incident wave—the eigenstates being parallel and perpendicular polarization with respect to the plane of reflection. In general the parallel polarization results in a larger shift.

An approximate argument which reproduces the physics and allows an approximate formula for the shift to be calculated can be made by considering only two plane waves. Let a plane wave of wave number k approach an interface at angle θ with respect to the normal so that the conditions for total internal reflection are satisfied ($\theta > \theta_c$). n_1 is the index of the medium in which total internal reflection takes place, n_2 is that of the outside (Fig. 1). In the x direction, the initial wave is given by

$$E_i = \exp(ik_x x).$$

Reflection produces a phase change δ in the wave that is a function of angle and of the boundary conditions at the

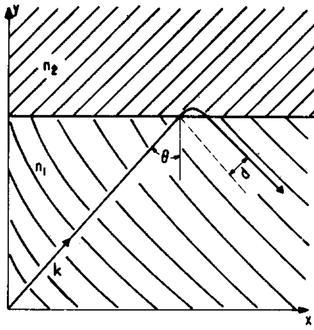

Fig. 1. Coordinate system used in the article.

interface. δ is given by the Fresnel equations and its derivation is in all standard optics texts. Thus the reflected wave is given by

$$E_R = \exp\{i[k_x x + \delta(\theta)]\}.$$

If one thought the wave had reflected with no phase change, one would ascribe an apparent origin of the wave behind the interface if phase information in the wave were measured.

Now consider a second plane wave, out of phase by π from the first, and at a very slightly different angle θ' such that $\theta + \Delta\theta = \theta'$ where $\Delta\theta$ is small, and also $k_x' = k_x + \Delta k_x$. The initial and reflected second wave projected on the x axis is given by

$$E_i' = \exp[i(k_x' x + \pi)],$$
$$E_R' = \exp\{i[k_x' x + \pi + \delta(\theta')]\}.$$

Note that the superposition of the two initial waves produces a dark shadow approximately along the common direction of propagation. The two reflected waves when superimposed give

$$E_R^{\mathrm{TOT}} = E_R + E_R'$$
$$= \exp\{i[k_x x + \delta(\theta)]\}(1 + \exp\{i[\Delta k_x x + \Delta\delta(\theta) + \pi]\}),$$

where we have used $\Delta\delta(\theta) = \delta(\theta') - \delta(\theta)$. The shadow occurs at the point where

$$\Delta k_x x + \Delta\delta(\theta) = 2\pi m \quad (m = 0, 1, 2, \text{etc.})$$

This is to be compared with the case of reflection from a perfect conductor when $\Delta\delta(\theta) = 0$, which would give

$$\Delta k_x x_c = 2\pi m,$$
$$x - x_c = (1/\Delta k_x)\Delta\delta(\theta);$$

but $\Delta k_x = k \cos\theta \Delta\theta$, which gives

$$\Delta x = x - x_c = \frac{1}{k\cos\theta}\frac{\Delta\delta(\theta)}{\Delta(\theta)}.$$

The displacement of the shadow is $D = \Delta x \cos\theta$,

$$D = \frac{1}{k}\frac{\Delta\delta(\theta)}{\Delta\theta}$$

which becomes

$$D = \frac{\lambda}{2\pi}\frac{\partial\delta(\theta)}{\partial\theta}. \qquad (1)$$

This expression first derived by Artmann[7] is a good approximation to several more exact calculations. The interested reader should consult the excellent article by Lotsch (Ref. 3) on this subject.

The advantage of the Artmann expression for the Goos–Hänchen shift D is that it displays the physics of the situation in a straightforward way, namely, as follows.

(a) The longitudinal shift of a beam of radiation is proportional to the wavelength and to the first derivative with respect to angle of the phase change of a plane wave at reflection.

(b) Beam shifts can be measured on total internal reflection as compared to reflection from a (nearly) perfect conductor.

(c) These shifts are sensitive to the polarization of the initial beam since $\delta(\theta)$ is a function of the initial polarization of the wave.

(d) In this approximation for small change of phase with angle, the displacement is independent of beam shape. We would anticipate exactly at total internal reflection where $\partial\delta(\theta)/\partial\theta$ has a discontinuity, the simple expression (1) would not be valid.

By use of the Fresnel formula for δ, and by further approximating $\sin\theta \approx n_2/n_1$ when it is a multiplicative factor only, we can derive

$$d_\perp \cong \pi^{-1} n\lambda(\sin^2\theta - n^2)^{-1/2},$$
$$d_\parallel \cong d_\perp/n^2, \qquad (2)$$
$$n \equiv n_2/n_1.$$

These expressions are to be compared to the exact calculations of Renard,[8] who finds

$$d_\perp = \frac{\lambda}{\pi}\frac{\sin\theta\cos^2\theta}{\pi\cos^2\theta + \sin^2\theta - n^2}(\sin^2\theta - n^2)^{-1/2},$$
$$d_\parallel = \frac{\lambda}{\pi}\frac{n^2\sin\theta\cos^2\theta}{n^4\cos^2\theta + \sin^2\theta - n^2}(\sin^2\theta - n^2)^{-1/2}. \qquad (3)$$

In Fig. 2 we show the numerical values of the shift (for $n = 1/1.5$, $\lambda = 1$) for both the exact and approximate formula. Although the approximate formula is the more instructive, we find that for angles comparable to those encountered in our experiment, the differences are large compared to the exact expressions. We shall use only the exact expressions in comparing theory with data.

EXPERIMENT

We used a 3-cm microwave transmitter coupled via a waveguide to a rectangular horn antenna to produce the incident polarized beam. A similar antenna was connected to a receiver, which travelled on a mechanical translation device perpendicular to the expected beam shift. The

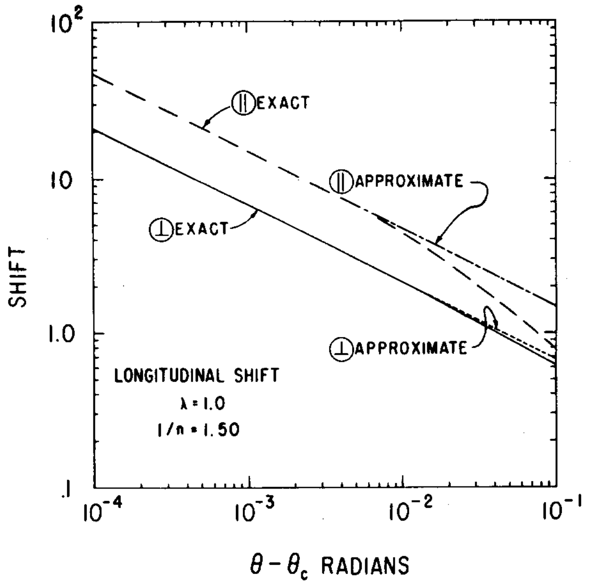

Fig. 2. The longitudinal shift of a very narrow beam at angle θ, encountering total internal reflection, as a function of θ. Exact refers to the exact expressions of Renard, while Approximate refers to the expressions of Artmann, evaluated for the case of parallel and perpendicular electric field vector with respect to the plane of reflection. The theory is evaluated for $\lambda = 1.0$ and $1/n = 1.50$.

beam was totally internally reflected in a paraffin prism of index 1.49 ± 0.01 measured also at 3 cm (see Fig. 3). The index was determined by two different techniques: phase change in a thick slab oriented perpendicular to the beam, and bend angle in the prism itself. Both methods agreed within errors. The wavelength was determined to be 3.20 ± 0.01 by repeated measurement of standing waves. Data runs were made under four different sets of conditions: parallel and perpendicular electric field vector orientation with respect to the plane of reflection, and with or without an aluminum plate at the interface whose presence destroyed total internal reflection and gave reflection from a nearly perfect conductor.

The longitudinal shifts were measured by subtracting the location of the peaks obtained with and without the aluminum plate. The data were recorded from the receiver output by a chart recorder whose time base was calibrated in terms of the horn position. We made a series of runs for each polarization state in which we varied in turn a number of parameters that should in principle not alter the data. These included distance of the horns from the prism, amount of microwave-absorbing material used on the nonreflecting face of the prism, etc. We also determined the peak position from the chart recorder by several methods. None of the systematic equipment changes nor data analysis techniques altered the final data in any significant way and the final results represent an overall average (see Table I).

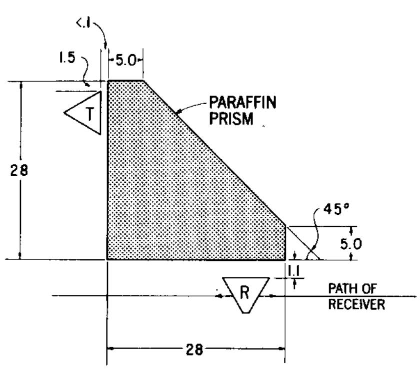

Fig. 3. Experimental setup—all linear dimensions are in cm. T is the transmitting horn, and R the receiving horn.

Table I. Results of the experiment.

Run	$d_\perp$(cm)	$d_\parallel$(cm)	$d_\parallel/d_\perp$
1	2.95	5.48	1.90
2	2.62	4.83	1.80
3	3.00	5.20	1.70
4	3.12	5.66	1.82
5	3.24	5.92	1.83
average	2.99 ± 0.20	5.42 ± 0.40	1.81 ± 0.10
theory	3.10 ± 0.20	5.30 ± 0.50	1.75 ± 0.10

RESULTS

Data were compared to the theoretical formulas of Renard, which yield an exact calculation of the longitudinal shift. Figure 2 shows the shift expected as a function of angle. The microwave antenna produced a relatively broad beam over a range of angles; thus the Renard formulas for the shift were averaged over the beam acceptance. Figure 4 shows the shifts expected on the basis of theory for a beam of unit intensity from θ_c to θ which is a good approximation to the actual beam used in the experiment.[9] Experimental values for the longitudinal shift were obtained from Fig. 4 by using $\theta = 0.15$ rad, which was measured for the equipment used. Final results are shown in Table I. These show rather good agreement with the theory.

CONCLUSIONS

We have designed a straightforward experimental arrangement capable of reasonably accurate measurements in the transverse shift of radiation at total internal reflection. We have presented a simplified argument which predicts that the general effect will occur and gives its magnitude to reasonable accuracy. Evaluation of the exact

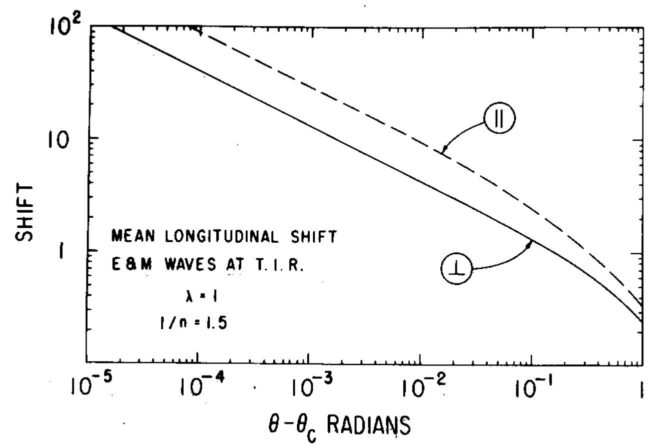

Fig. 4. The mean longitudinal shift for a wide uniform beam of radiation between θ and θ_c as a function of θ evaluated by using the exact expressions only—see Fig. 2 for explanation of other symbols.

Renard formula and averaging over a wide beam configuration are also presented. The data we have taken are to our knowledge only the third set of measurements of this phenomenon, and the only set of measurements which verify that the effect does occur on a single reflection. Furthermore, our measurements do not suffer from any possible effects due to incorrect assumptions about the phase changes at metallic reflectance since the metal used is much more nearly a perfect conductor at microwave frequencies. We have verified, in a larger angular region than has heretofore been investigated, that the shift agrees with theory. Finally, we have verified that the broad-beam configuration of a simple microwave transmitter can be used to demonstrate the effect. A short report of this experiment has already appeared in the literature.[10]

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NATURAL SELECTION

I always know when contributions come through the letter box that if I get no prefix or suffix [on my name] they are likely to be acceptable; "Esq." means somebody else has rejected them; "Dr." or "Professor" means that they have been rejected more than once; while both together mean that this is the continually orbiting paper from the Middle East or the Middle West explaining what is wrong with relativity.

—G.R. Noakes, Editor, *Contemporary Physics*