Microwave Experiments and Their Optical Analogues by Gorden Ferrie Hull, Jr.

Appendix J, Concepts of Classical Optics, John Strong

Appendix J

Microwave Experiments and Their Optical Analogues

by Gordon Ferrie Hull, Jr.†

The advent of vacuum tubes for the production of microwaves and of fixed crystals for detecting them has made it possible to demonstrate readily all the optical properties of electromagnetic radiation. The apparatus which is described makes use of radiation of wavelength 3.2 cm, and the experiments which will be carried out are essentially those for free space microwaves, demonstrating the analogous phenomena of geometrical and physical optics. These experiments have been partially described in a previous paper.[1]

Because a microwave generator produces plane polarized, coherent, monochromatic radiation, one would expect some differences to occur between the optical properties of free space microwaves and those of light. Such differences do occur, for a light source is seldom strictly monochromatic; its wavelength is less by a factor of 10^5, and its radiation is not coherent. Consequently, the optical analogues of microwaves must be considered as analogues and not as identities.

Generator and Receiver for 3.2 cm Microwaves

The generator and receiver for 3.2 cm microwaves are shown in Fig. J-1. A Western Electric 2 K 24 or 723 A/B reflex klystron is used as the microwave source.[2] This tube requires two power supplies, one regulated at 300 volts and about 30 ma., to accelerate the electron stream through the cavity, and another to apply a variable negative voltage from 0 to −300 volts between the cavity and the repeller. A means of modulating the repeller voltage with

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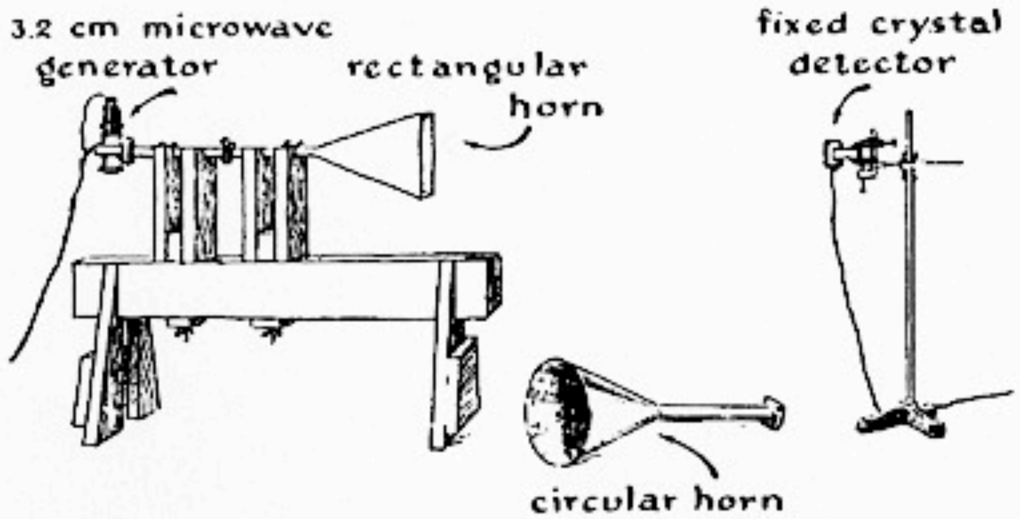


FIG. J-1 Transmitter and receiver with horn radiators for 3 cm microwave experiments.

an audio oscillator should also be provided. This tube will deliver about 30 mw. power, in a frequency range from 8500 to 9700 mc. (from 3.5 to 3.1 cm). The coaxial output of the 723 A/B tube is coupled to a standard 0.5″ × 1″ (outside dimensions) rectangular wave guide having a wall 0.056″ thick. The TE_{01} mode[3] is excited in the rectangular guide, and the microwaves are propagated along it and radiated from the 20 db horn, which is shown in Fig. J-1 mounted together with the 723 A/B tube and its section of wave guide on a wooden bench. Wave guide mounts for the 723 A/B tube and also power supplies can be bought from suppliers of microwave apparatus or can be made. The details of construction of the wave guide mount are given in the Western Electric circular describing the operation and performance of the 723 A/B tube.

The receiver, which is shown clamped to a rod stand in Fig. J-1, consists of a short section of wave guide into which a Western Electric 1N23B fixed crystal detector is appropriately mounted. This piece of wave guide is shorted at one end and has a plane rectangular coupling flange at its open end. The crystal output is connected by coaxial microphone cable to either an audio amplifier and loudspeaker for demonstration or to a vacuum tube voltmeter if precision measurements are required. If the crystal is "square law," as it usually is, the vacuum tube voltmeter will measure directly the relative microwave power received. Although such a receiver can be built, it is simpler to buy a wave guide crystal mount from one of the suppliers of microwave apparatus. Since the receiver is small, it can be used as a probe for exploring radiation coming from different directions. It can be held in the hand for this purpose and moved about. Since the TE_{01} mode has the electric

vector across the short dimension of the wave guide, the receiver is also an analyzer for the polarization of the microwave radiation. Finally, the receiver can be mounted on a wooden bench similar to the one on which the transmitter is mounted. Other sections of wave guide and horns can be attached to the receiver by means of the coupling flange.

In Fig. J-1 two horn radiators are shown: a rectangular one attached to the generator on the wooden bench and a circular one in the foreground.[4] Each horn has an absolute gain of 20 db. The rectangular horn, which is attached to a standard 0.5″ × 1″ rectangular guide, has the dimension of $3.6\lambda \times 4.45\lambda$ for its open end and an axial length of 6λ where λ is the free space wavelength. The circular horn has a diameter of 4.4λ for its open end and an axial length of 6λ, and is attached to a standard 1″ (outside diameter) circular wave guide having a wall 0.032″ thick. Each of these horns has a total beam width of about 8° at the half-power points. The circular horn can be substituted for the rectangular horn on the transmitter, and the TE_{11} mode[5] will be excited in the circular wave guide. If this substitution is made, a standing wave will be produced in the section of rectangular guide from the generator because of the sharp discontinuity at the rectangular-circular wave guide junction. For demonstration purposes the discontinuity is not troublesome. For many measurements, however, it is desirable to eliminate the standing wave. This can be done by the insertion of a transition section of guide, which changes gradually from rectangular to circular wave guide. Such a transition section is shown in Fig. J-2.

A number of other wave guide components, also shown in Fig. J-2, are useful in wave guide measurements. At the bottom of Fig. J-2 is a 0.5″ (internal diameter) circular guide loaded with a polystyrene rod. When the polystyrene rod is removed, the wave guide diameter is below cutoff, and hence the microwaves are not transmitted through it. Next is the transition section from rectangular to circular guide, followed by a twist section to change the polarization through a right angle, and, last, two rectangular wave guide bends. With the apparatus shown in Figs. J-1 and J-2, the experiments with wave guides previously described by the author for 10 and 20 cm microwaves can be performed with 3 cm microwaves.[6]

Transmission and Reflection

Besides the transmission of 3 cm microwaves through the various wave guide components shown in Fig. J-2, the transmission of free space microwaves through various dielectrics such as sheets of glass and plywood can be demonstrated by inserting the dielectrics between the transmitter and the receiver. These dielectrics are also partial reflectors. For these demonstra-

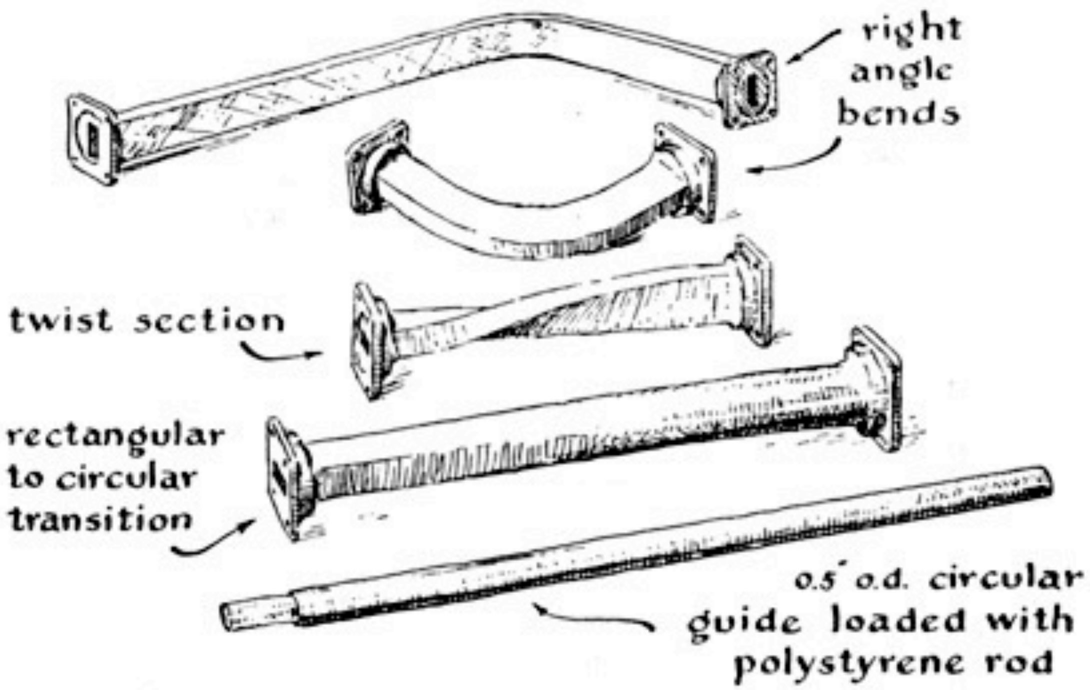


FIG. J-2 Wave guide components for demonstrating transmission of 3 cm microwaves through wave guides.

tions the receiver should be at some distance from the horn radiator. A sheet of copper and $\frac{1}{8}''$ mesh copper screen are excellent reflectors and prevent the transmission of the 3 cm microwaves. Standing waves in air can be produced by reflection from a copper sheet and an approximate wavelength measurement made. A plywood sheet which has been wound with wire spaced $\frac{1}{8}''$ apart completely stops and reflects 3 cm microwaves when the wires are parallel to the electric vector, but transmits the radiation when the wires are at right angles. If the wire is spaced 0.5″ on the plywood sheet, it will be found to be about half reflecting and half transmitting when the wires are oriented parallel to the electric vector.

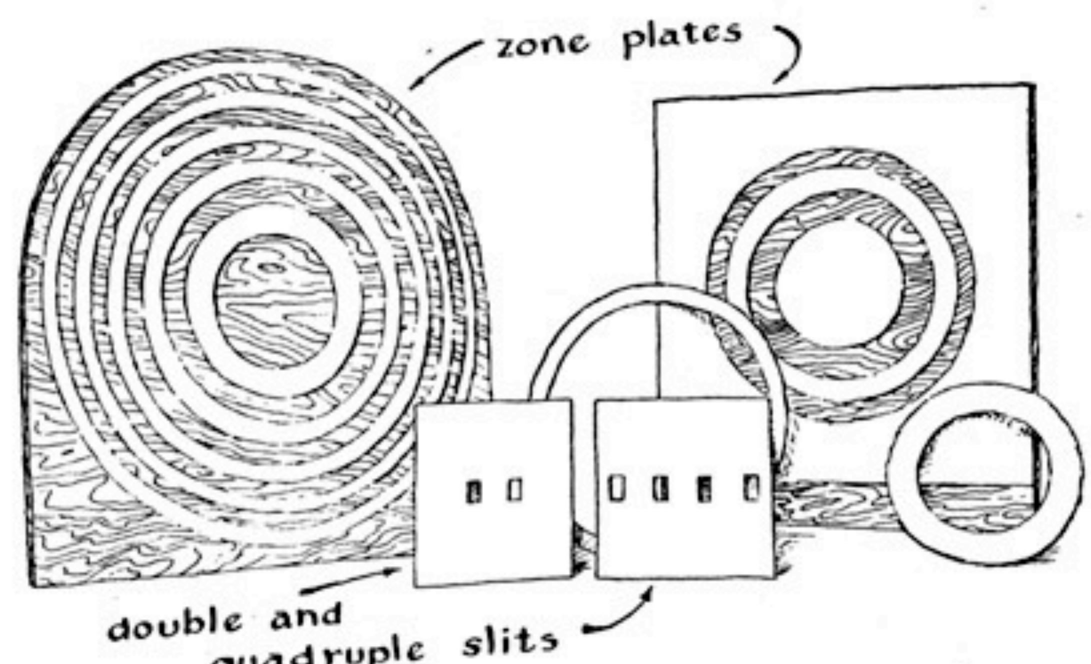
Instead of using a sheet of brass as a reflector, one can also use an ordinary plane mirror of silvered glass. It is easy to show for microwaves as for light that the angles of incidence and reflection are equal. Two silvered mirrors or two brass sheets at right angles will reflect the microwave radiation in the direction from which it came. Finally, a concave spherical mirror, if large in aperture, will focus the microwave radiation sharply. The author uses a concave mirror, of silvered glass, 12″ in diameter and 20″ in focal length, for this purpose. For short focal lengths and large diameters, parabolic reflectors of metal are used.

Interference and Diffraction

Because of the long wavelength of microwaves, compared with light, interference from double or multiple slits can be demonstrated with large slits spaced only a few wavelengths apart. In Fig. J-3 two brass plates 10″ square are shown with two and four slits. The slits are $\lambda/2 \times \lambda$ in size and are 2λ apart. By making the slits narrow, the diffraction pattern due to a single slit covers a total angle of more than 180° and has little effect on the interference pattern produced by the slits. To show the interference from two slits, the brass plate is placed in a holder clamped to the wooden bench supporting the transmitter, a few inches from the open end of the horn. The receiver can then be moved about in front of the double slit screen to locate the maxima. Because the slit spacing is 2λ, there are maxima at 30° and 90° on each side of the central maximum. A brass plate with two slits spaced 4λ apart will give four maxima on each side of the central maximum. Finally, the brass plate shown with four slits spaced 2λ apart gives an interference pattern which is the combination of the two double slit patterns in which the slit spacings are 2λ and 4λ.

To measure the interference pattern from double slits quantitatively, the apparatus is arranged as shown in Fig. J-4. The receiver, clamped to a rod stand, is fastened with a wire 5–6′ long to the base of the wooden holder

FIG. J-3 Double and quadruple slits and zone plates for demonstrating interference and diffraction of 3 cm microwaves.



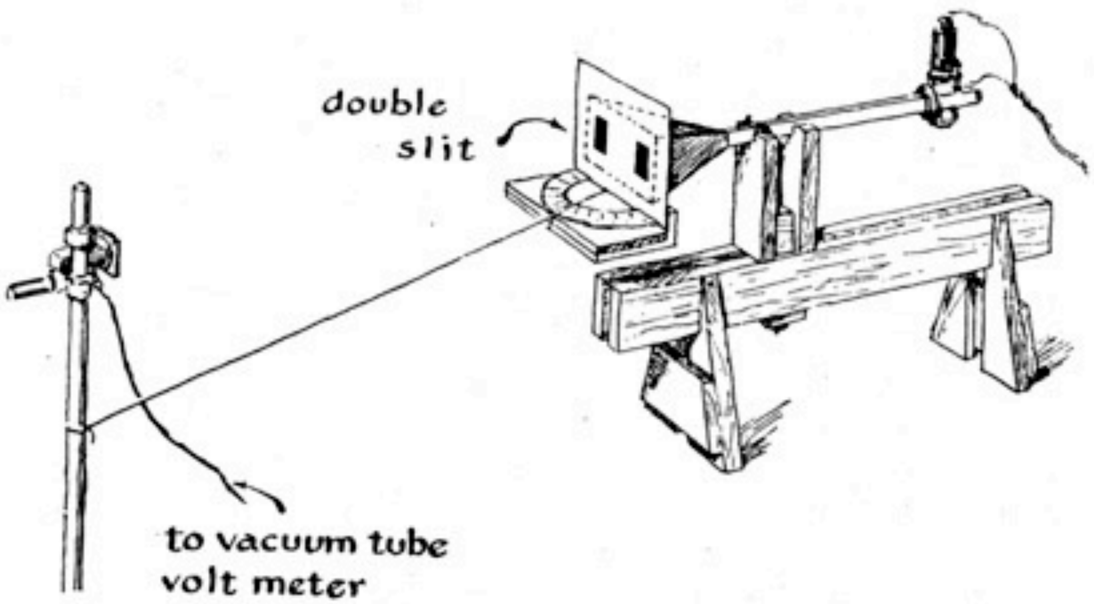
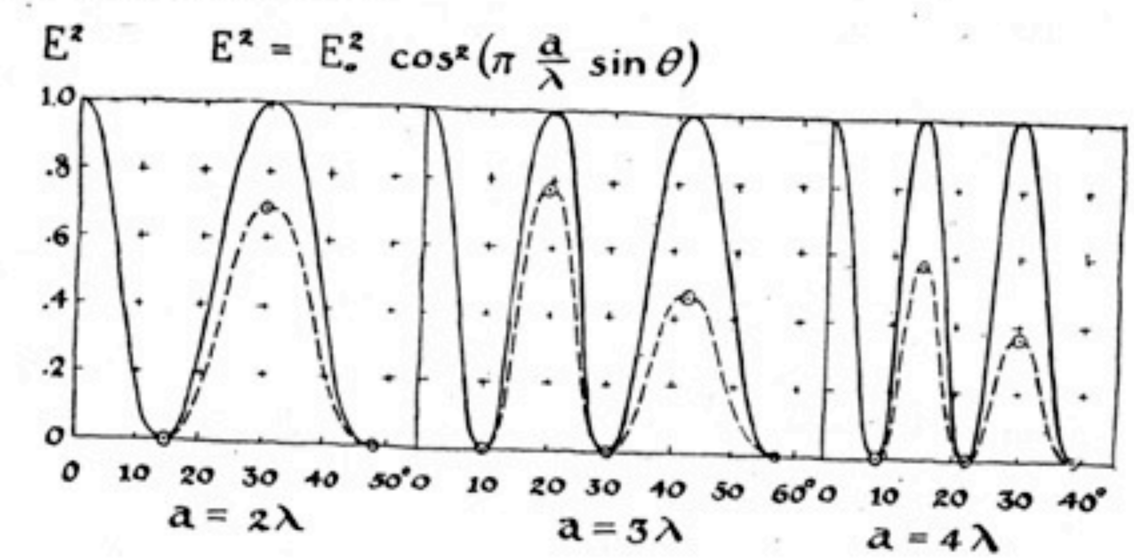


FIG. J-4 Apparatus for measuring interference patterns from double slits.

supporting the brass plate containing the double slit. A large protractor is also fastened to the wooden holder just below the wire, so that, as the receiver is moved along the arc of a circle, whose radius is the length of the wire, the angle which the receiver makes with the axis of the double slit can be measured with the protractor. The output of the receiver is connected to a vacuum tube voltmeter. In Fig. J-5 are shown quantitative measurements of interference maxima and minima from double slits spaced 2λ, 3λ, and 4λ apart. The solid curves are the theoretical interference patterns. It will be noted that the experimental measurements of maxima and minima occur at the correct angles but that the intensities of the maxima decrease with increasing

FIG. J-5 Quantitative measurements of interference patterns from double slits spaced 2λ, 3λ, and 4λ apart.

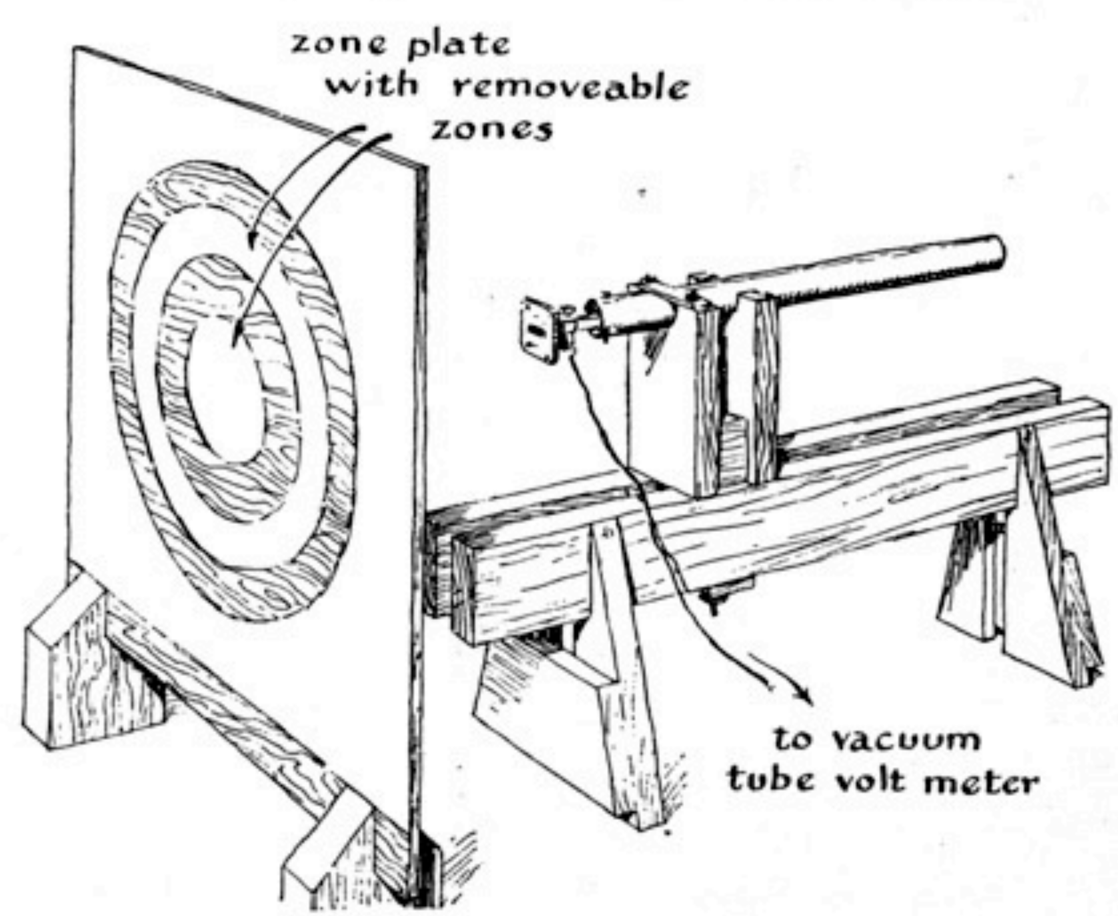


angle. This is due in part to the fact that the diffraction pattern from a single slit is not uniform with angle and decreases in intensity slowly with increase in angle from the central axis.

As a corollary to Young's double slit experiment, interference can be produced by means of reflection as in Lloyd's single mirror experiment. All that is required is to place a brass plate just to one side and in front of the horn radiator and then investigate the interference pattern with the receiver.

In Fig. J-3 two zone plates are shown for demonstration of Fresnel diffraction. The fixed zone plate on the left has even-numbered half-period zones cut from galvanized sheet iron and tacked to plywood 0.5″ thick, thus exposing the odd-numbered, half-period zones. The zone plate on the right has four zones, cut from galvanized sheet iron, which are supported on a plywood board with two pins at the top of each zone. These zones are made removable so that the effect of removing successive zones one after another can be demonstrated when the receiver is placed at the focal point of the zone plate. Each zone plate has a focal length of 10λ, and the radii of the zones are given by the usual equation $r_n = [nf\lambda + (n\lambda/2)^2]^{1/2}$, where n is the number of the zone, f the focal length, and λ the free space wavelength. The zone plates should

FIG. J-6 Removable zone plate for measurement of Fresnel diffraction.

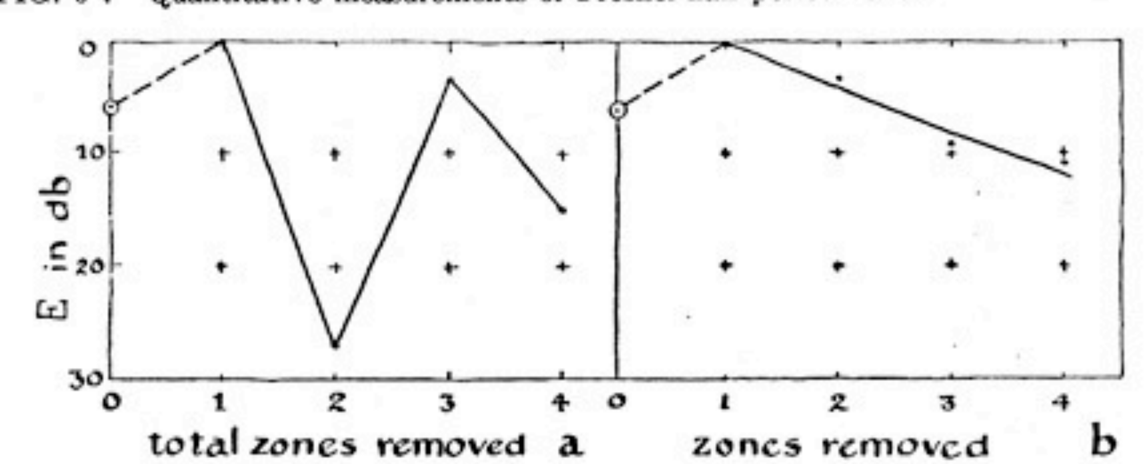


be placed at some distance from the transmitter to ensure that a plane wave will strike the zone plate. When this is done, the focal point is very sharp and can be easily located within $\pm\lambda/2$.

To measure the removable zone plate quantitatively, the apparatus is arranged as shown in Fig. J-6. The removable zone plate is supported in front of a wooden bench, and the receiver, connected to a vacuum tube voltmeter, is mounted on the wooden bench in an appropriate holder at a distance from the zone plate equal to the plate's focal length, 10λ. First the illumination of the signal $\frac{1}{2}c\kappa_0 E_0^2$ from the transmitter is measured in the absence of the zone plate, and this value is used for reference. After the zone plate is placed in front of the bench, successive zones are removed one at a time, and the illumination $\frac{1}{2}c\kappa_0 E^2$ of the received signal is measured. In Fig. J-7a the ratio in db, $10 \log_{10} \frac{E^2}{E_0^2} = 20 \log_{10} \frac{E}{E_0}$, is plotted against the number of zones removed. When the first half-period zone is removed, the received amplitude, compared with the received signal in the absence of the zone plate, is doubled (increased 6 db). Removal of the second zone decreases the received signal by 27 db (a factor of 1 to 22.5 in amplitude), signifying almost complete cancellation of the amplitude from the first by that from the second. Removal of the third zone increases the signal again, and removal of the fourth zone decreases the signal, as expected, but does not produce complete cancellation. The reason for this is shown in Fig. J-7b, in which successive zones are removed. The fact that the straight line is not horizontal shows that the amplitude from each zone is not constant. This is due to the fact that the amplitude of the wave front over the whole aperture of the zone plate is not constant, being largest at the center and slowly decreasing radially outward, as would be expected from the transmitter horn diffraction pattern.

Fraunhofer diffraction from a rectangular or circular opening is exhibited by the radiation patterns of a rectangular or circular horn or of a parabolic

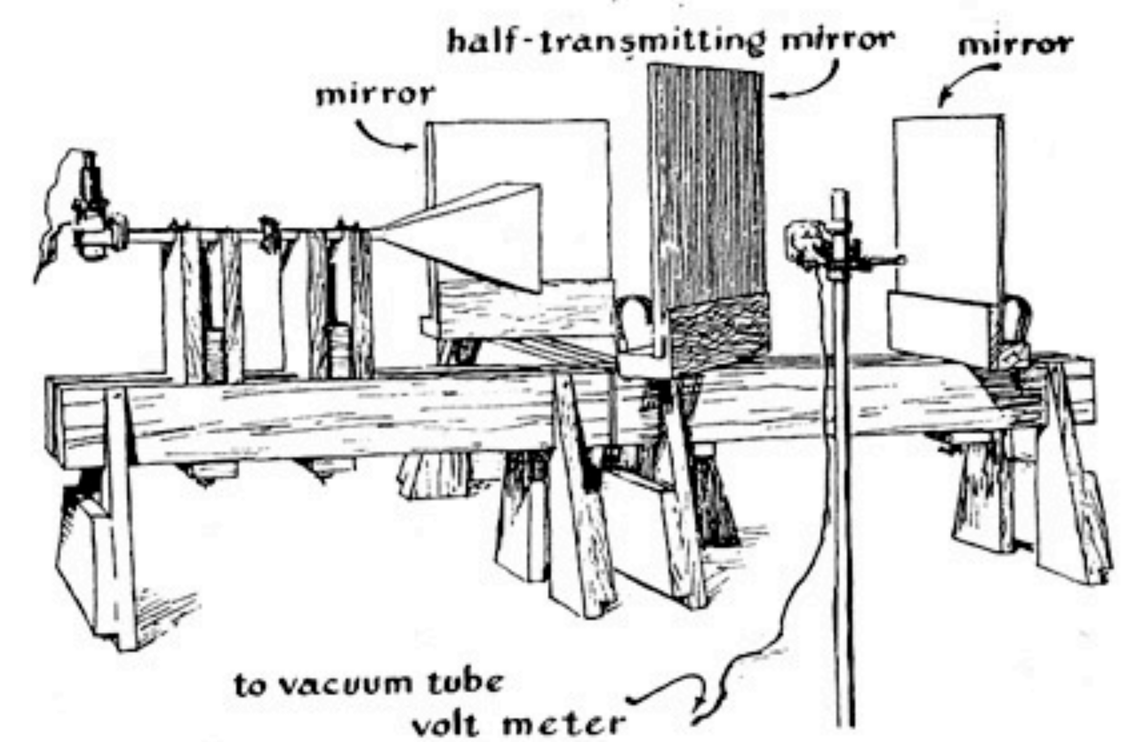
FIG. J-7 Quantitative measurements of Fresnel half period zones.



reflector.[7] To demonstrate or measure a Fraunhofer diffraction pattern, a horn with receiver attached should be mounted on a rotating stand at a large distance from the microwave transmitter. The transmitter and the horn whose pattern is to be measured should first be lined up, and then, as the horn is rotated, the received microwave power as a function of angle is measured. The vacuum tube voltmeter connected to the crystal detector of the receiver will measure relative microwave power directly, provided the crystal obeys the square law. To obtain the diffraction pattern of a parabolic reflector, the receiver should be mounted at the focus of the parabola and the entire assembly rotated about a vertical axis, as is done with the horn.

Another interesting demonstration, as well as an instrument for precision measurement, is the microwave Michelson interferometer. This instrument, shown in Fig. J-8, consists of the microwave transmitter; two totally reflecting mirrors of brass 10″ square, mounted on movable supports on the wooden benches; a half-reflecting mirror made by winding wires 0.5″ apart on a 12″ plywood board, which is mounted on a rotating support on a wooden bench; and the receiver, which is shown clamped in a rod stand in the foreground. The transmitter, mirrors, and receiver must be carefully lined up. When one of the totally reflecting mirrors is moved slowly along the wooden bench, the receiver will indicate the passage of maxima and minima corresponding to bright and dark fringes in the optical case, as shown in Fig. J-9. From precise measurement of the distance between minima or maxima the

FIG. J-8 Michelson interferometer for 3 cm microwaves.



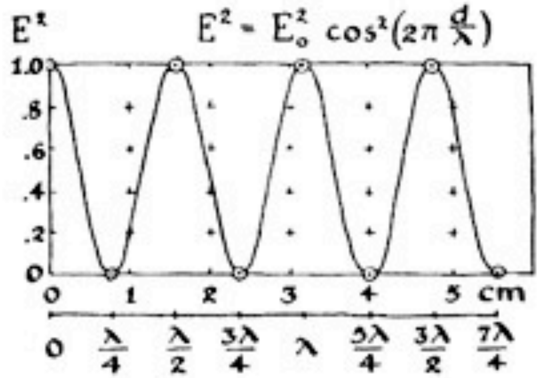


FIG. J-9 Measured and calculated Michelson interferometer fringes.

wavelength of microwaves can be obtained, as in the optical case. In fact, when the interferometer is properly adjusted, the minima are extremely sharp and are from 30 to 40 db below the maxima in intensity. If several sheets of dielectric such as glass or plywood are placed in one arm of the interferometer, the receiver will indicate maxima and minima as the sheets are removed one after another. The index of refraction of a dielectric can be measured by noting the fringe shift produced by the insertion of a known thickness of the dielectric in one of the interferometer arms, as is done in the optical case. In the microwave region the index of refraction of ordinary window glass is about 2.0 and that of plywood about 1.3. Hence, four sheets of single weight window glass or four sheets of $\frac{1}{4}''$ plywood inserted in one arm of the interferometer will produce a shift of about one fringe. The microwave Michelson interferometer is capable of high precision and is especially useful in measuring the dielectric constants of artificial microwave dielectrics, which will be discussed in the next section and which cannot be placed inside a 3 cm wave guide. As a precision instrument, the interferometer must be rigidly constructed with rigid mirrors equipped with screw drives.

Refraction, Total Internal Reflection, and Artificial Dielectrics

The refraction of microwaves by dielectric materials can be demonstrated in many ways. It is possible to use, as the refractive medium, ordinary matter which may or may not be transparent to light; or, because of the special properties of microwaves, artificial dielectrics which refract microwaves but not light can be constructed. Two types of artificial dielectrics will be discussed.

In Fig. J-10 two 60° prisms, 10″ on a side, and one right angle prism are shown. The two prisms on the left are made of paraffin (index of refraction 1.47), and the one on the right is made of sheets of galvanized iron forming parallel-plate wave guides. The paraffin prisms are contained in forms made of $\frac{1}{4}''$ plywood. If the 60° paraffin prism is placed in front of the horn radiator,

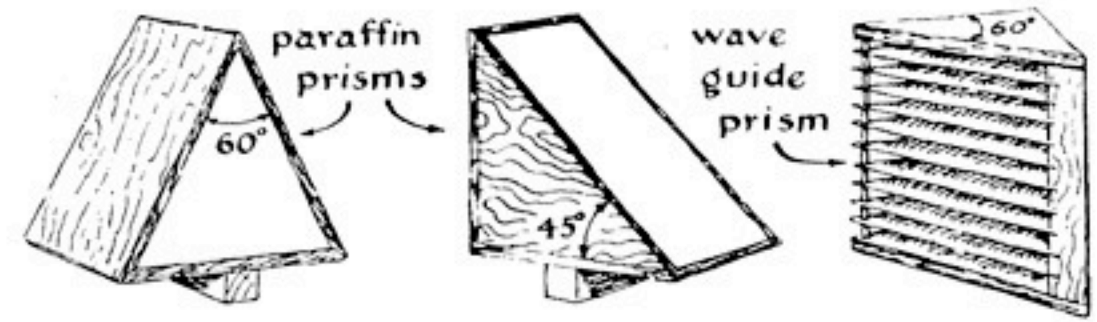


FIG. J-10 Paraffin and wave guide prisms for 3 cm microwaves.

it is found that the microwave beam is bent in the same way that a light beam is bent by a glass prism. Holding the microwave receiver in the hand and rotating the prism, one can locate the angle of minimum deviation, which for a 60° paraffin prism is 33°. With a right angle prism made of paraffin, total internal reflection of the 3 cm microwave beam can be demonstrated. If the microwave receiver is brought close to the totally reflecting surface of this prism, the presence of surface waves, the distance by which these waves emerge from the paraffin surface, and their polarization can be readily measured.

The expression for the amplitude of the surface wave emerging from the totally reflecting surface of the paraffin right angle prism[8] is

$$E_y = E_{oy} \exp 2\pi j \left(ft - \frac{x \sin r}{\lambda}\right) \exp\left(-\frac{2\pi z}{\lambda}\sqrt{N^2 \sin^2 i - 1}\right)$$

where $j = \sqrt{-1}$, f is the frequency, t the time, λ the wavelength, i the angle of incidence, r the angle of refraction, and N the index of refraction; the coordinate x and y axes are those indicated in Fig. J-11. The first exponent represents the propagation constant and the second the attenuation constant. This wave is traveling along the surface in the x direction. Its wave front, or surfaces of constant phase (yz plane), is at right angles to the surfaces of constant amplitude (yx plane). According to the second exponential term, the amplitude of the surface wave is damped out exponentially as z increases. As indicated in the upper right-hand part of Fig. J-11, the rate at which the surface wave is damped out as z increases can be measured. The measurements are shown in Fig. J-11 for a paraffin right angle prism which gives a slope of 5 db/cm (or 1.79/cm). Using the second exponential term in the equation, the expression $20 \log_{10} \frac{E_y}{E_{oy}}$ is calculated with $\lambda = 3.2$ cm, $N = 1.47$, and $i = 45°$. This gives a slope of 5.2 db/cm (or 1.82/cm), which is to be compared with the experimental measurement of 5 db/cm.

The wave guide prism in Fig. J-10 operates only as a prism when the electric vector is parallel to the metal plates of the prism. For this case the TE_{01}

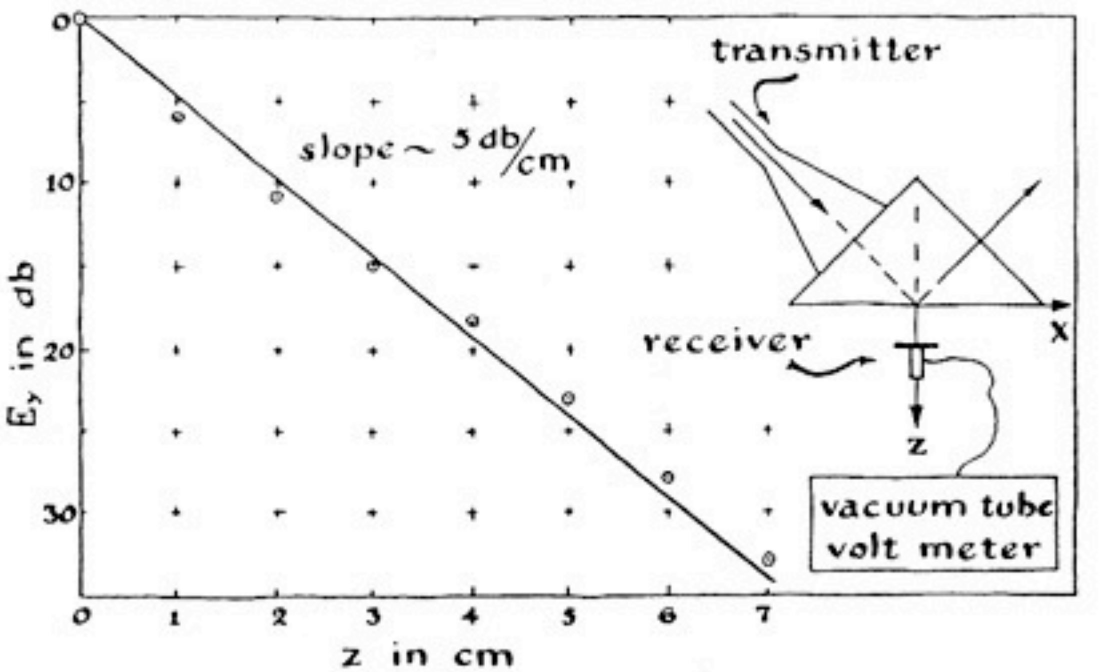


FIG. J-11 Measured damping of the emerging wave at the surface of a totally reflecting right angle paraffin prism.

mode for a parallel plate wave guide is excited, and the wave velocity in the parallel plate guide is greater than the free space velocity. The wave velocity is given by $v = c[1 - (\lambda/2b)^2]^{-1/2}$, where c is the velocity of light, λ the free space wavelength, and b the spacing between the metal plates.[3] The index of refraction of the parallel plate wave guide is then $N = [1 - (\lambda/2b)^2]^{1/2}$. The prism shown has an index of refraction of 0.6, corresponding to a plate spacing of $b = 0.79''$. The plate spacing is critical and should be maintained to within ± 2 percent. Because the index of refraction is less than unity, the microwave beam is bent in the opposite direction from what it is for the paraffin prism. By rotating the wave guide prism, an angle of minimum deviation can be located with the aid of the receiver; it is $-24°$. If the wave guide prism is oriented with its plates perpendicular to the electric vector, the TM_{00} mode, whose wave velocity is the same as in free space,[3] is excited, and consequently the microwave beam is not deviated. The artificial wave guide dielectric prism therefore operates only for plane polarized microwaves with the electric vector parallel to the wave guide plates. Evidently this dielectric will exhibit a type of double refraction for unpolarized microwaves, and later we shall describe experiments in which this property of the metal plate dielectric is used.

Besides prisms, lenses also can be constructed. Plano-convex lenses of paraffin can easily be made by filling watch glasses of 8″ or 10″ diameter with paraffin. Glass lenses and lenses made of artificial dielectrics can also

FIG. J-12 Three lenses for 3 cm microwaves. From left to right are a glass lens, an artificial disk dielectric lens, and a wave guide lens.

be used. In Fig. J-12 three lenses are shown. The lens at the left is a 10″ glass, plano-convex, condensing lens; the lens in the center is an artificial dielectric lens made up of an array of thumb tacks; and the one on the right is a lens made up of parallel-plate wave guides. All of these lenses exhibit the usual properties expected of lenses. Their focal length for 3 cm microwaves can be determined experimentally within $\pm\lambda$, which, considering that the lens diameters are about 8λ, is reasonable precision.

The parallel-plate wave guide lens has the same refractive properties as the wave guide prism previously described. The galvanized iron lens plates are supported in a wooden frame 12″ × 12″ with a plate spacing of $b = 0.79''$ ± 2 percent, which gives an index of refraction of 0.6. Since the index of refraction is less than unity, a converging lens is plano-concave. Such a lens has a focal length of 12λ, a diameter of 9λ, and a radius of curvature of 4.8λ. With these particular dimensions, it is not necessary to zone or step the lens, and the departure of the spherical surface from the true ellipsoidal surface is not greater than 0.2″, or a phase difference of $\lambda/16$ at the extreme, which is within the tolerance limits for this type of lens.[9] To obtain the correct radius for each plate, it is simpler to draw the entire lens to scale and take off the radii with dividers rather than calculate each radius individually. Like the wave guide prism, the wave guide lens operates as a lens only when the electric vector is oriented parallel with the plates.

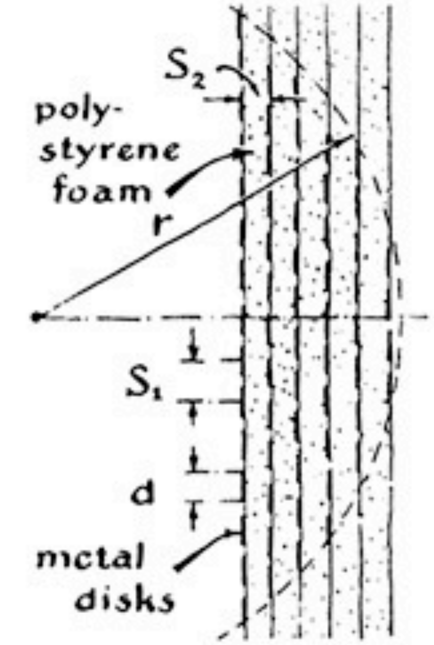
It is interesting to note at this point that we can define an index of refraction for a parallel-plate wave guide from which we can make the same type of calculations as for an ordinary dielectric. For example, we can calculate the reflection coefficient. Also, we can define a characteristic wave impedance for the parallel-plate wave guide and for free space, and from these quantities calculate the reflection coefficient. Both methods must yield the same value for the reflection coefficient, and consequently one would expect analogies to exist between transmission line theory and optics.[10] Another example of this analogy is the quarter-wave transmission line and coated lens. Two transmission lines of different characteristic impedances can be connected, without producing reflection, by a third transmission line, a quarter wavelength long, whose characteristic impedance is the geometric mean of the characteristic impedances of the two lines. Similarly, a glass lens coated with a quarter-wavelength thickness of dielectric whose index of refraction is the geometric mean of the indices of refraction of glass and free space will be reflectionless for one particular wavelength.

Another interesting analogue is the microwave equivalent of molecular arrays, which is exemplified by the lens made of thumb tacks shown in the center of Fig. J-12. Since a piece of metal whose dimensions are small compared with a wavelength can be driven in forced oscillation by an electromagnetic radiation field, an array of identical metal pieces such as spheres, disks, or rods should behave in the same way that a dielectric made up of a molecular array behaves when exposed to light. In other words, an array of disks have a dielectric constant $x_e = 1 + \mathfrak{N}\mathfrak{p}/\kappa_0$, where $\mathfrak{p}$ is the polarizability of the disk, $\mathfrak{N}$ the number of disks per unit volume, and κ_0 the electric inductive capacity of free space. This is the same as the classical expression for a dielectric in which $\mathfrak{p}$ is the polarizability of a molecule and $\mathfrak{N}$ the number of molecules per unit volume.

The expression for x_e is applicable only when the disks are far from resonance and there is no interaction between the fields of adjacent disks. In general, we would expect an artificial disk dielectric to obey the Clausius-Mossotti equation at long wavelengths and to exhibit the phenomenon of anomalous dispersion at short wavelengths when the microwave frequency approaches the resonant frequency of the metal disks.

The general criterion for the design of an artificial dielectric made up of an array of identical metal elements is that the dimensions of the elements should be less than $\lambda/4$ and the spacing of the elements less than λ. If the spacing is greater than λ, diffraction, similar to X-ray diffraction by crystals, occurs. Furthermore, the metal elements should be thin in the direction of propagation of the microwaves.[11] On the basis of this criterion, the artificial disk dielectric lens shown in the center of Fig. J-12 was constructed. Since the polarizability of a metal disk is $\frac{2}{3}\kappa_0 d^3$, where d is the disk diameter, the dielectric constant or square of the index of refraction of a disk dielectric is $N^2 = x_e = 1 + \frac{2}{3}\mathfrak{N}d^3$. The actual array used is shown in Fig. J-13, and the dimensions shown are $d = 1$ cm, $s_1 = 1.3$ cm, and $s_2 = 1$ cm. These dimensions give $\mathfrak{N} = 1.18$ disks per cm³ and a calculated index of refraction $N = 1.33$. The metal disks used were thumb tacks $\frac{3}{8}''$ (0.95 cm) in diameter, which were stuck into sheets of polystyrene foam 1 cm thick and 8″ square. Poly-

FIG. J-13 Arrangements of disks in artificial disk dielectric lens for 3 cm microwaves.



styrene foam has a density of about 1.5 pounds per cubic foot and an index of refraction of 1.01. It has practically no refractive effect on microwaves. The lens is plano-convex with a radius of curvature of 5″ and a calculated focal length of 15″. Five sheets of polystyrene foam are used. As shown in Fig. J-13, the disks in alternate layers are staggered. This is done in order to increase the number of disks per cm³. To determine the radius of the circular area to be covered by thumb tacks on each sheet of foam, it is simpler to draw the lens to scale and take off the radii with dividers rather than calculate each radius individually. The positioning of the thumb tacks is best accomplished by marking out the circular area and dotting in the thumb tack centers on thin paper. This paper is then placed on the foam sheet and the thumb tacks pushed through the paper into the foam. After all the sheets have been filled with the required number of tacks, the sheets are put together and supported in a wooden frame. The lens, when completed and measured, is found to have a focal length of 10″ instead of the calculated value. This means that the index of refraction of the disk dielectric is 1.5 instead of the calculated value of 1.33. The discrepancy between calculated and measured index of refraction is to be expected because of the effect of the Clausius-Mossotti equation and because the diameter of the thumb tacks is slightly larger than $\lambda/4$. Unlike the wave guide dielectric lens, the disk dielectric lens operates independently of the polarization of the microwave radiation. Also, the index of refraction of the disk dielectric—but not of the wave guide dielectric—remains essentially constant for longer wavelength microwaves.

The index of refraction of artificial dielectrics can be measured with high precision with the microwave Michelson interferometer discussed in the previous section. The procedure is the same as in optics: one inserts a sheet of artificial dielectric about 1′ square and of known thickness in one arm of the interferometer and measures the fringe shift, from which the index of refraction can be calculated.

Polarization

As has been pointed out, the microwave radiation from the transmitter is plane polarized. For polarization experiments it is often desirable to have elliptically or circularly polarized radiation. Elliptically polarized radiation is easily obtained by placing a sheet of glass, polystyrene, or other dielectric in front of the horn radiator, with the plane of the sheet at 45° to the electric vector and parallel to the direction of propagation. Elliptically and circularly polarized microwaves can also be obtained by use of the artificial wave guide dielectric discussed in the previous section. If this dielectric is made with a plate spacing to give an index of refraction of 0.6 when the electric vector is oriented parallel to the plates, it will also have an index of refraction of unity when the electric vector is at right angles to the plates. Consequently, if an appropriate thickness of wave guide dielectric is placed in front of the horn radiator with the plates oriented at 45° to the electric vector, elliptically, circularly, or plane polarized radiation will result. The thicknesses for a quarter- or half-wave plate of wave guide dielectric are calculated in the same way as in optics, using 0.6 for the extraordinary and unity for the ordinary index of refraction. These thicknesses are 0.79″ and 1.58″ for the quarter- and half-wave plates, respectively, and the thickness tolerance is ±2%. The plates are supported in a wooden frame 12″ square and have a plate spacing of 0.79″ ± 2 percent, the same spacing as the wave guide prism and lens. Although the quarter- and half-wave plates behave in a manner similar to that of those used in optics, the wave guide dielectric is not exactly similar in its double refracting properties to a uniaxial crystal, for the wave guide dielectric does not have an optic axis. In general, microwaves pass through the wave guide dielectric with two components, one of which travels faster than the other, and whose amplitudes depend upon the orientation of the electric vector of the incident microwave radiation with respect to the plates forming the wave guides.

One can extend the principle of the quarter-wave plate to a circular wave guide so that the radiation from a circular horn will be circularly polarized. All that is necessary is to split up the microwave radiation in a circular wave guide operating in the TE_{11} mode into two components of equal amplitudes and at right angles and to delay the phase of one component by $\lambda/4$ with respect to the other. This can be achieved by inserting in a circular piece of wave guide a sheet of dielectric along the diameter and oriented at 45° to

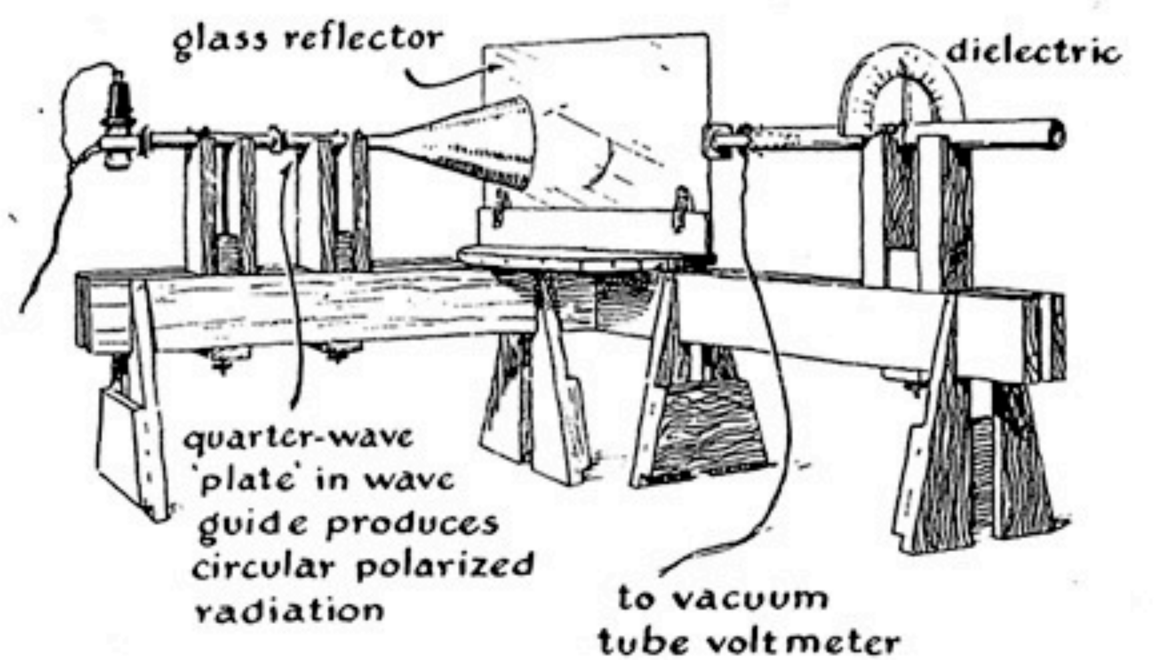


FIG. J-14 Transmitter and receiver arranged for producing plane-polarized 3 cm microwaves by reflection from glass at Brewster's angle.

the electric vector of the incident microwaves. The proper thickness and length of such a dielectric are not easily calculated but can be found experimentally. The horn radiator shown attached to the transmitter in Fig. J-14 has a polystyrene strip $\frac{1}{16}$″ thick and 2.5″ long across the diameter of the 1″ standard circular wave guide leading to the horn. The strip is oriented at 45° to the electric vector of the incident microwaves, and the horn radiates circularly polarized radiation. If this polystyrene strip is replaced by one of the same thickness and 5″ long, the result is a half-wave plate, and the radiation coming from the horn is plane polarized with the electric vector rotated 90°.

A number of interesting experiments can be performed with various types of polarized microwaves. Circularly polarized microwaves can be plane polarized by reflection from a dielectric at the Brewster angle. In Fig. J-14 the circular horn equipped with a quarter-wave plate radiates circularly polarized microwaves which are incident upon several sheets of window glass held in a rotating support equipped with a protractor on the wooden bench. Microwave radiation reflected and transmitted by the glass sheets is investigated with the receiver, which is mounted in a rotating holder on another wooden bench. A protractor is fastened to the rotating holder so that the polarization of the reflected and transmitted microwaves can be measured. When the glass plates are adjusted at the Brewster angle, the reflected microwaves are plane polarized, with the electric vector vertical, while the transmitted microwaves are generally elliptically polarized. The index of refraction of dielectrics can be measured by this method in the same way as in optics.

Experiments on the rotation of the plane of polarization of microwaves by sugar solutions, liquids such as turpentine, and crystals such as quartz have not shown any measurable rotation. However, it has been found that the Faraday effect exists in the microwave region for certain paramagnetic salts,[12] ferrites,[13] and plasma in electric discharges in gases.[14]

Evidently other experiments can be devised to show particular properties of microwaves. It has been the purpose of this appendix to describe a number of simple experiments demonstrating the properties of free space microwaves and the measuring techniques which are used in microwave research, and their similarity to optical experiments

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