ABSTRACT

Example Harmonic and Anharmonic Oscillators Demonstrated and Analyzed

I easily modified a commercially available teaching apparatus to make spring and magnetic “hardening” oscillators. That apparatus is demonstrated with position (time) data collected and graphed. Simple approximate fits reveal the character of their dissipations in addition to the anharmonic oscillator's expected period variation and transition to harmonicity. Phase plotting makes the latter especially obvious. I also numerically modeled the spring oscillator with a simple leapfrog algorithm using the force constant found experimentally.

Introduction

While studying the motion of a spring oscillator cart supported pendulum, I noticed the cart's interesting behavior when nearing the magnetic bumpers I had used as spring fixtures. I realized, with some improvement, I would have a useful hardening oscillator. After developing that apparatus, I made a similar Hookian one for comparison and presented both at the most recent SCAAPT meeting at CSUCI.

The Apparatuses

Both oscillators consist of low friction carts running on a track. The linear (spring) oscillator uses similar springs stretched between the cart and hooks in wood blocks clamped to adjustable brackets fastened to the track to supply the restoring force. The hardening (magnetic repulsion) oscillator uses cylindrical magnets mounted in the cart and in wood bumpers bolted to the track to supply its restoring force. The carts' masses plus bolted steel cubes supply the necessary inertias. The position of the carts is determined by a linearized rotary motion sensor (optical encoder). The RMS is fastened to a bracket bolted to the track. A heavy thread loop clamped to the cart, looped over the RMS's pulley, and a small pulley on a bracket at the opposite end of the track transmits the motion of the cart to the RMS. The two adjacent photographs illustrate the preceding.

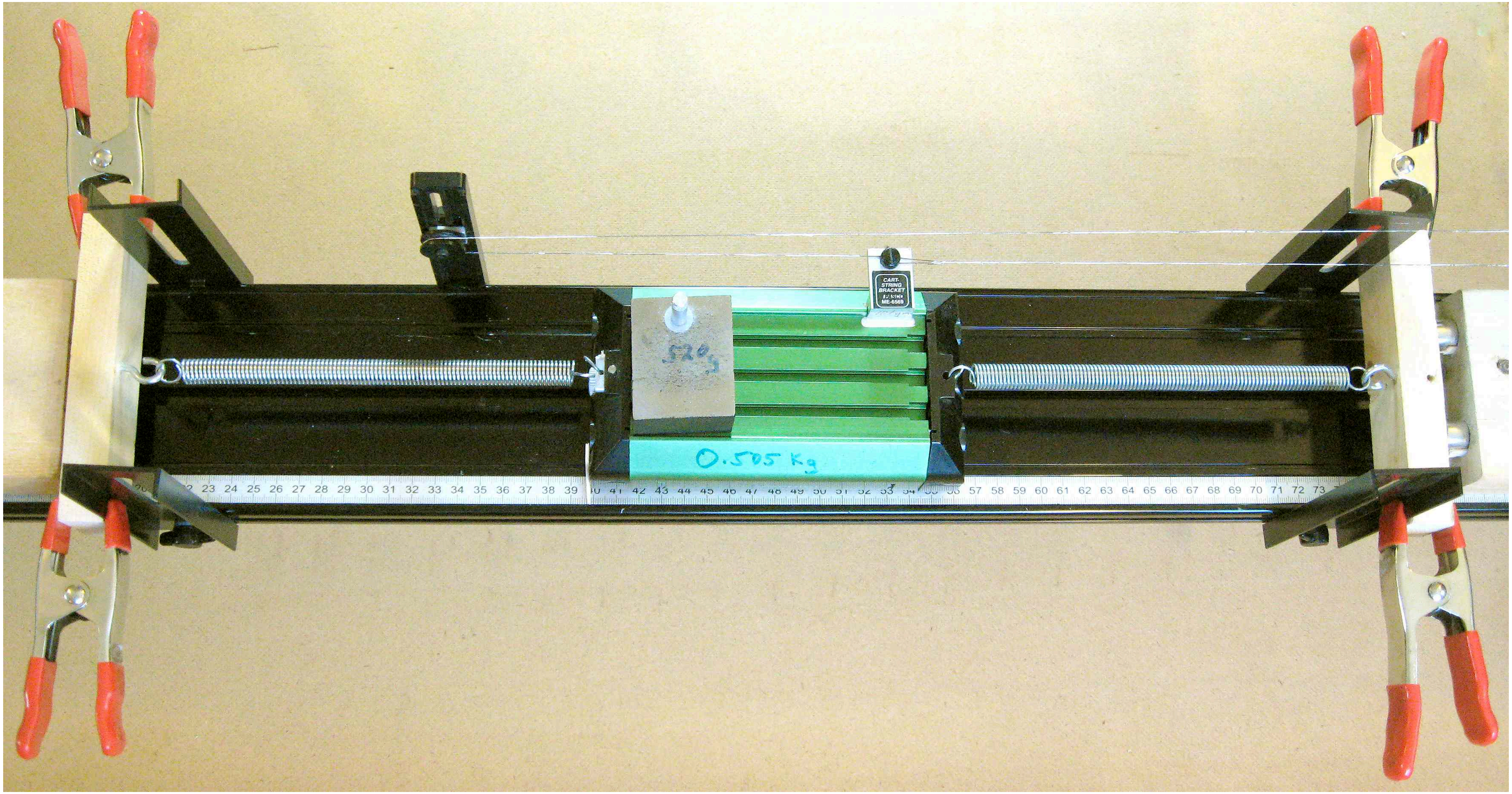


Plate One, The Hookian (linear spring) Oscillator

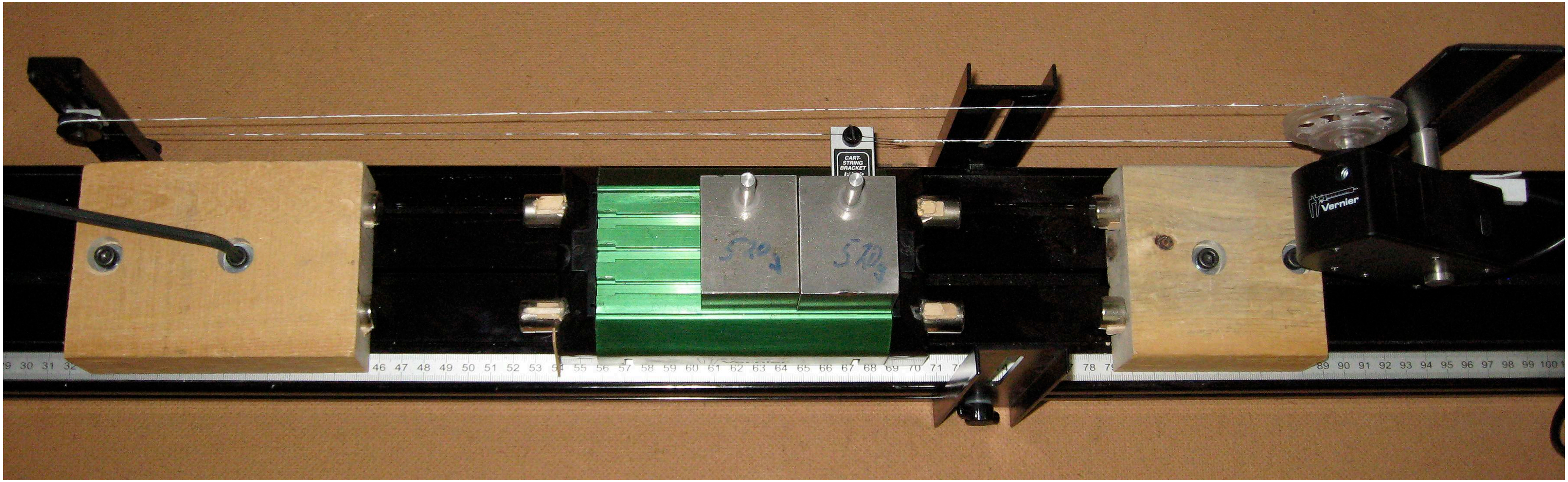


Plate Two, The Magnetic (non-linear hardening) Oscillator

Note: All of the commercial apparatus is from Vernier, except the cart clamp, pulley and its bracket, which are from Pasco. I salvaged the eight magnets from linear generator powered flashlights from RAFT. I shimmed the magnets to prevent their movement, because I'd used RTVR for easy removal.

Data Collection and Display

I used Vernier's LabQuest interface and LoggerPro for data collection, analysis, and display with one exception. Various plotted data sets adjacent illustrate typical behavior of the oscillators.

Notes

My intention was qualitative only, though one may compare, for example, the measured period and that calculated from the mass (Plate 3. 0.66 kg) and a spring constant of 243 N/m. See plate seven.

The fits shown in plate three reveal a likely, and expected, small proportion of Coulomb damping in addition to viscous, as shown by the better fit. In addition, as noted, the oscillator is not strictly linear, but a very slightly hardening one. Plate four shows the expected period change due to hard repulsion, and also suggests a dissipation mixture. To obtain the triple fits in plates five and six I exported the data to Kaleidograph, which is not limited to five fitting coefficients. I used a Vernier force probe to collect the data displayed in plate seven. Plate nine shows the algorithms I used for spring oscillator modeling with a resulting graph.

Conclusion

By stealing from the Freshman Lab. and simple addition and modification, I think, one may create apparatus suitable for an interesting study of oscillators at the intermediate lab. level.

Further

I intend to add turbulent damping to an oscillator and numerically model it, and to find the bugs in my coulomb dissipation and magnetic restoring force algorithms.

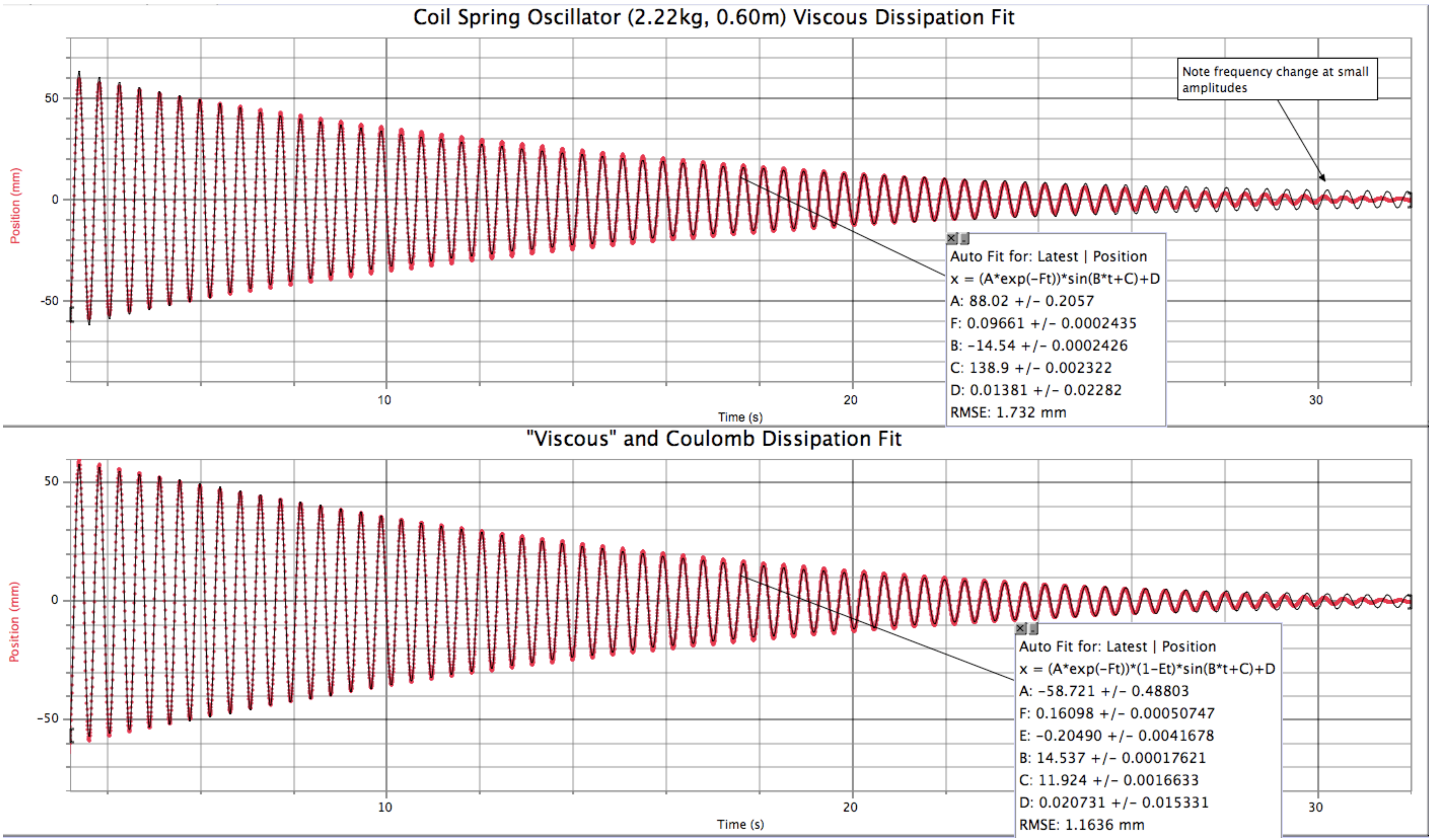
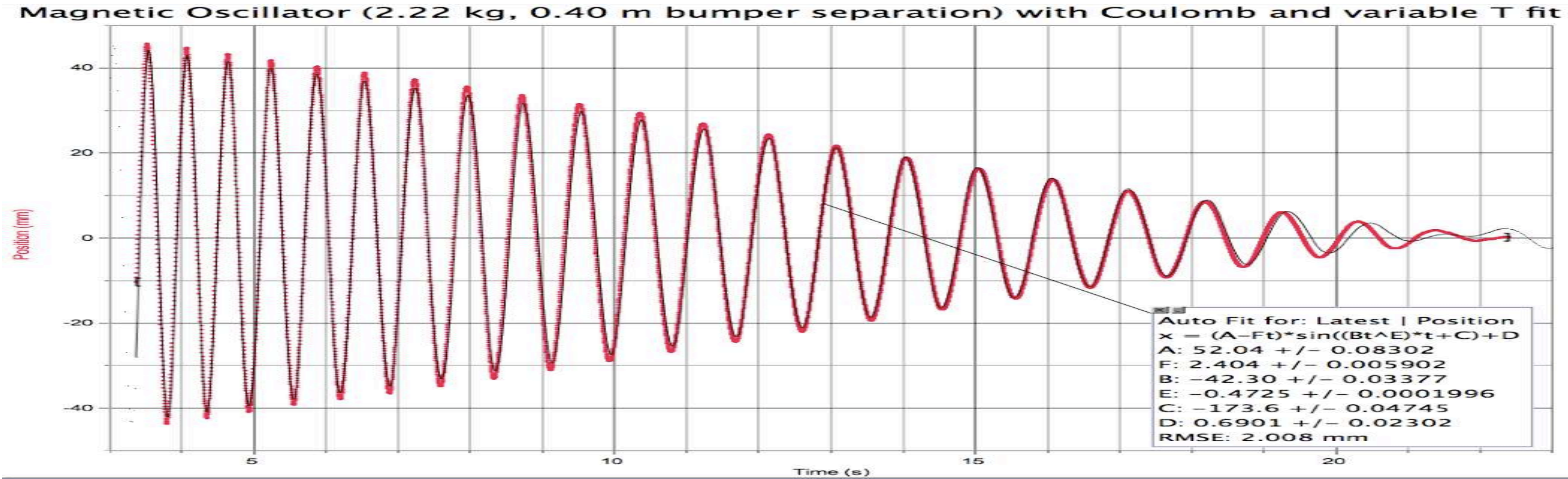


Plate Three, Spring Oscillator with Two fits



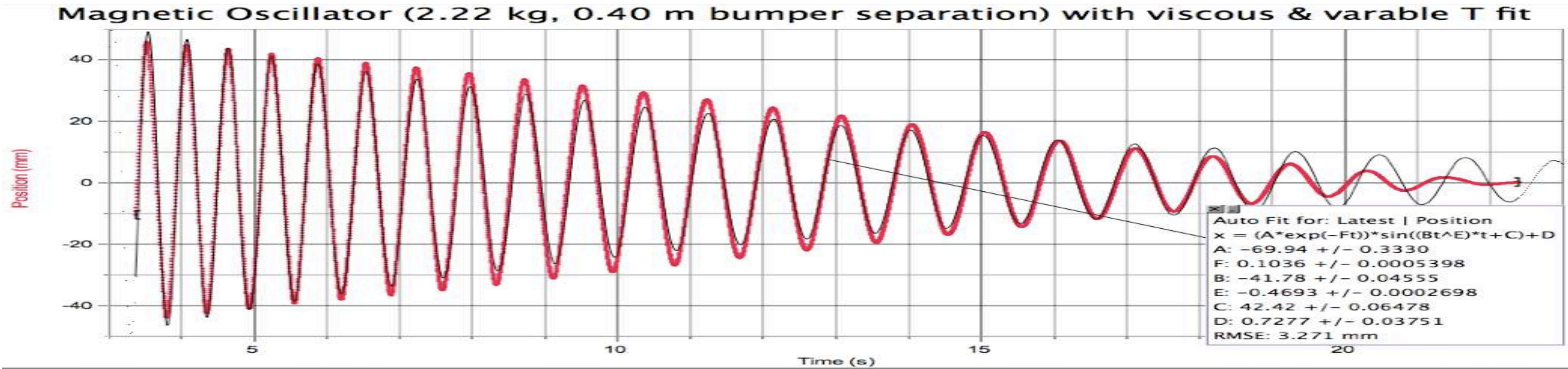


Plate Four, Magnetic Oscillator with Two fits

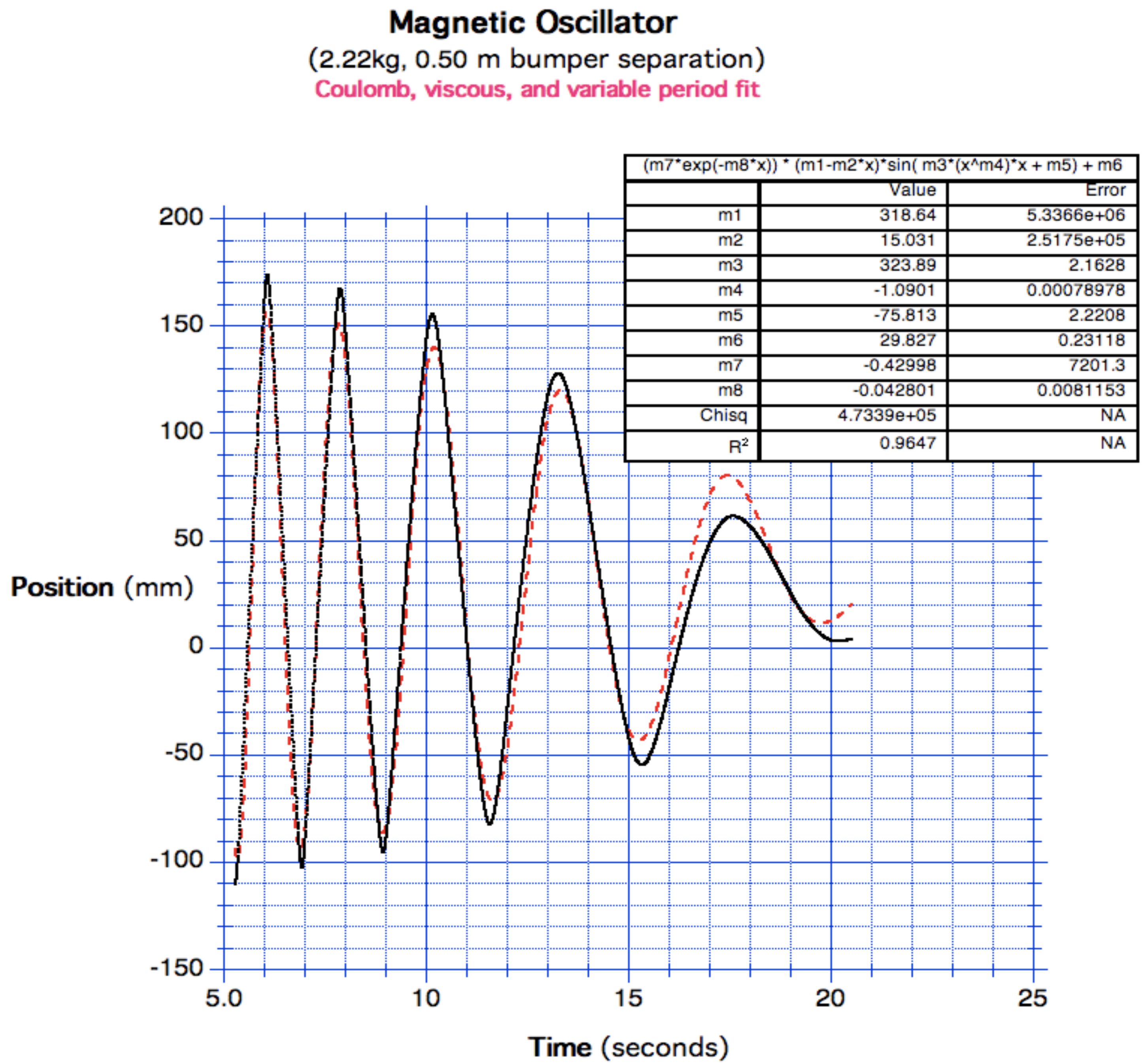
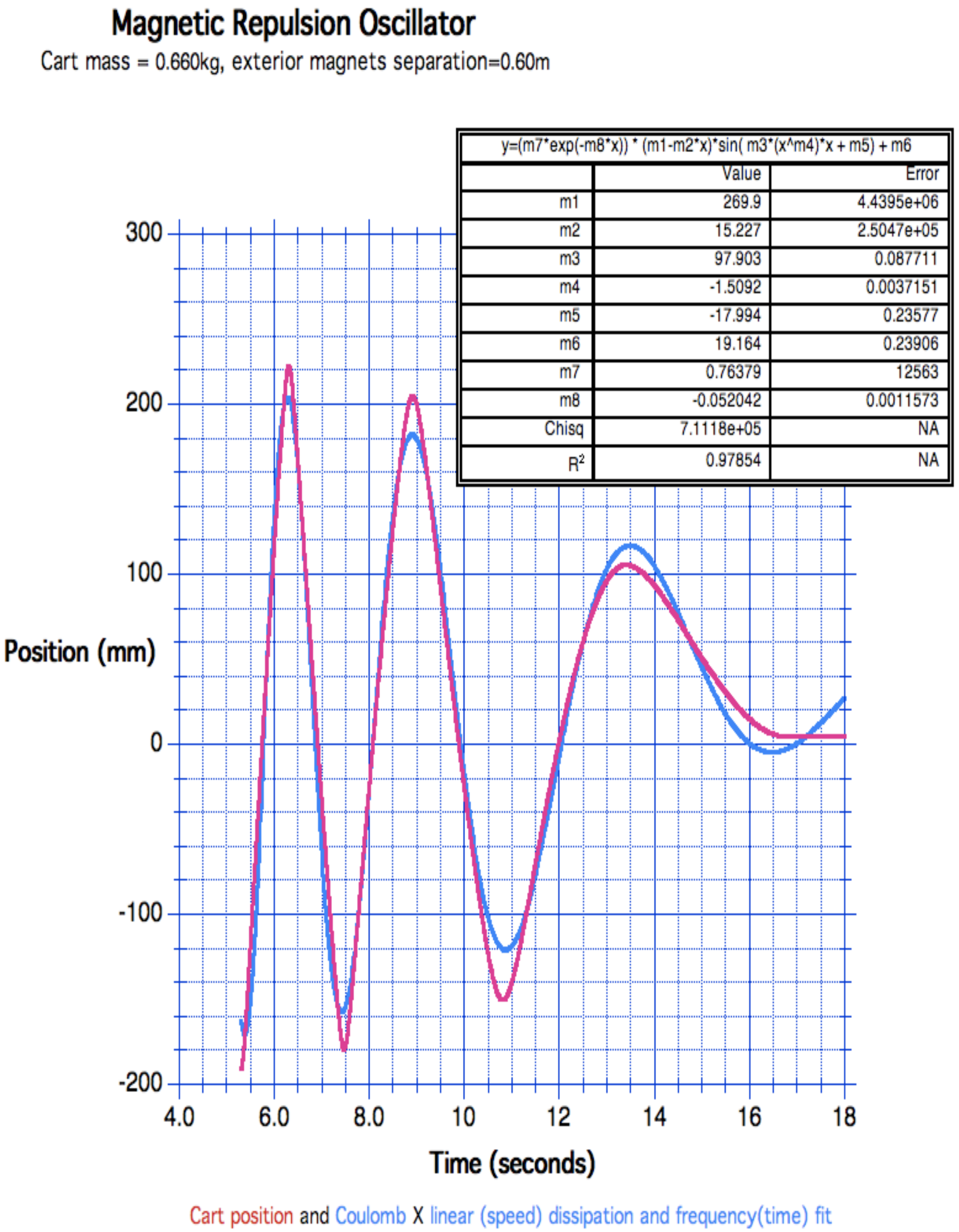


Plate Five, Magnetic Oscillator with Triple Fit



y=(m7*exp(-m8*x)) * (m1-m2*x)*sin(m3*(x^m4)*x + m5) + m6		
	Value	Error
m1	269.9	4.4395e+06
m2	15.227	2.5047e+05
m3	97.903	0.087711
m4	-1.5092	0.0037151
m5	-17.994	0.23577
m6	19.164	0.23906
m7	0.76379	12563
m8	-0.052042	0.0011573
Chisq	7.1118e+05	NA
R^2	0.97854	NA

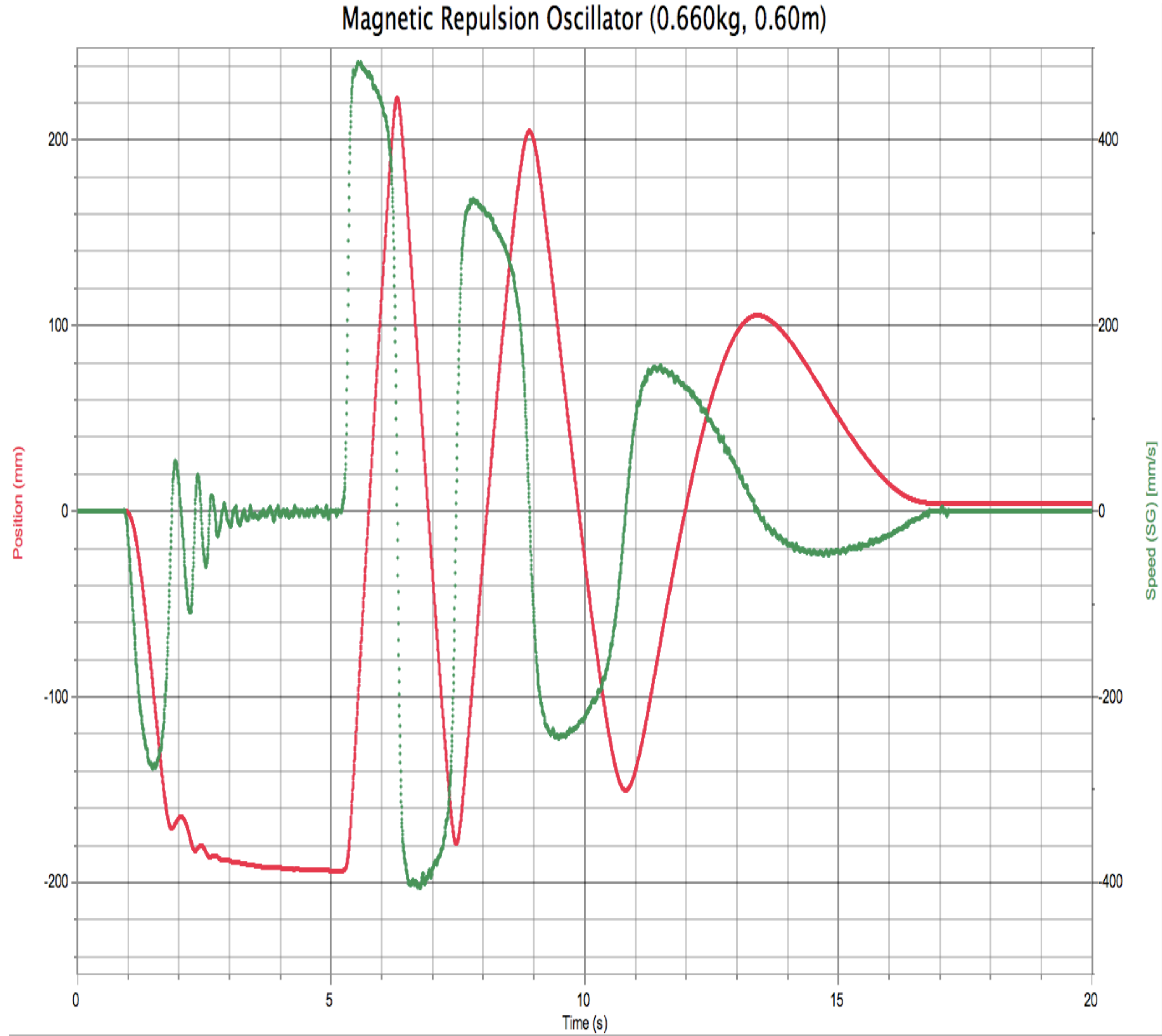
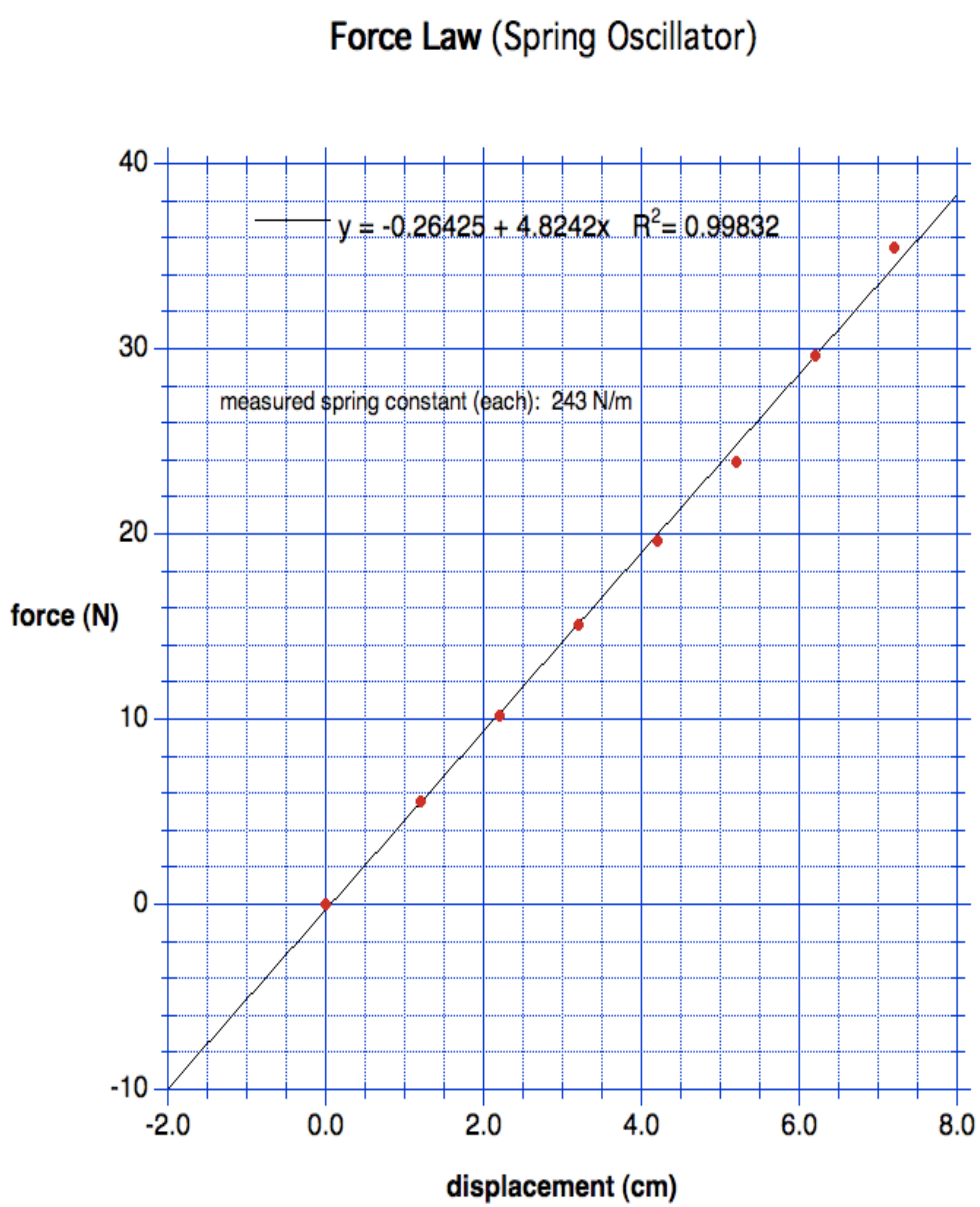


Plate Six, Magnetic Oscillator Position(time) and Speed



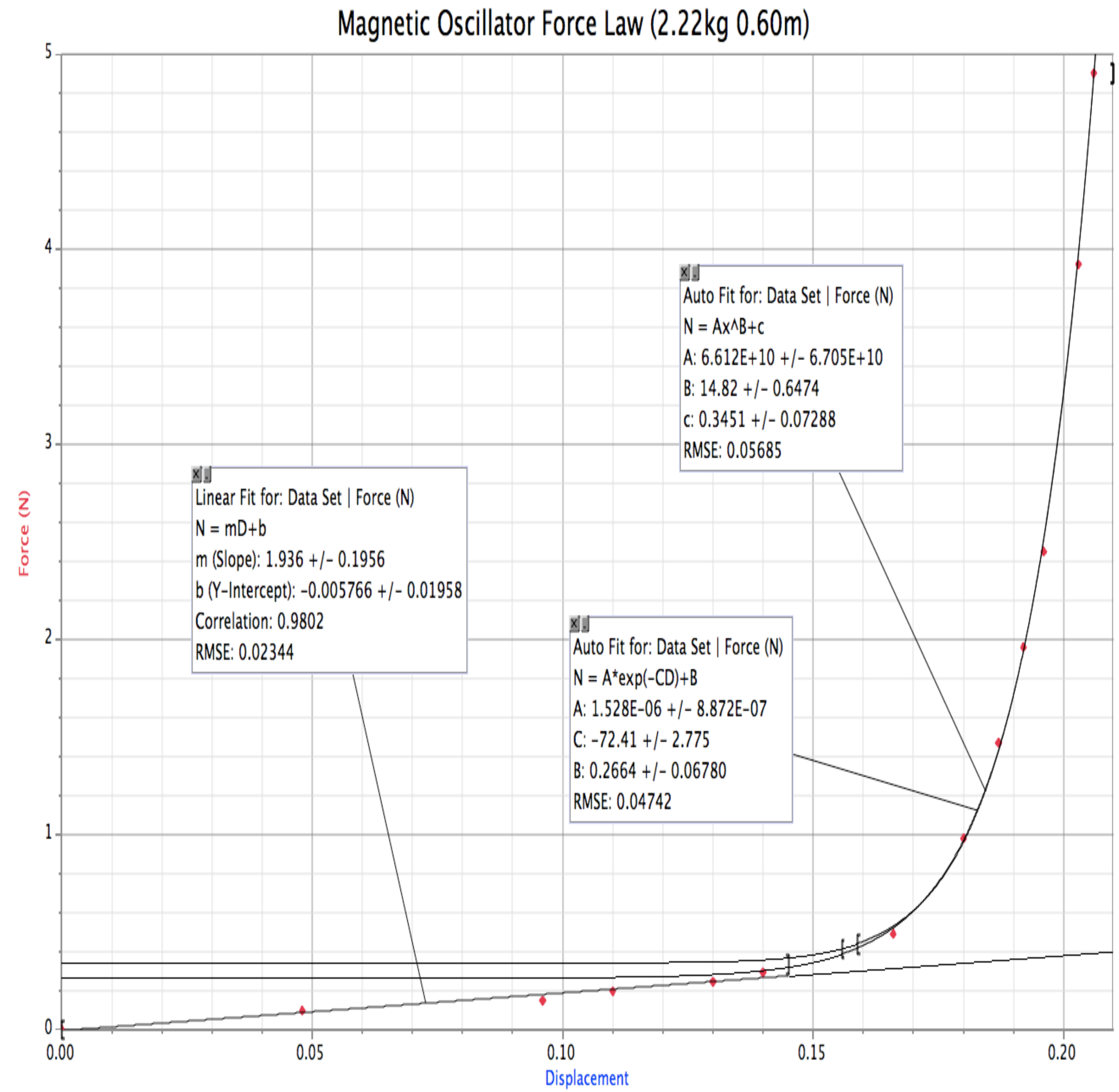
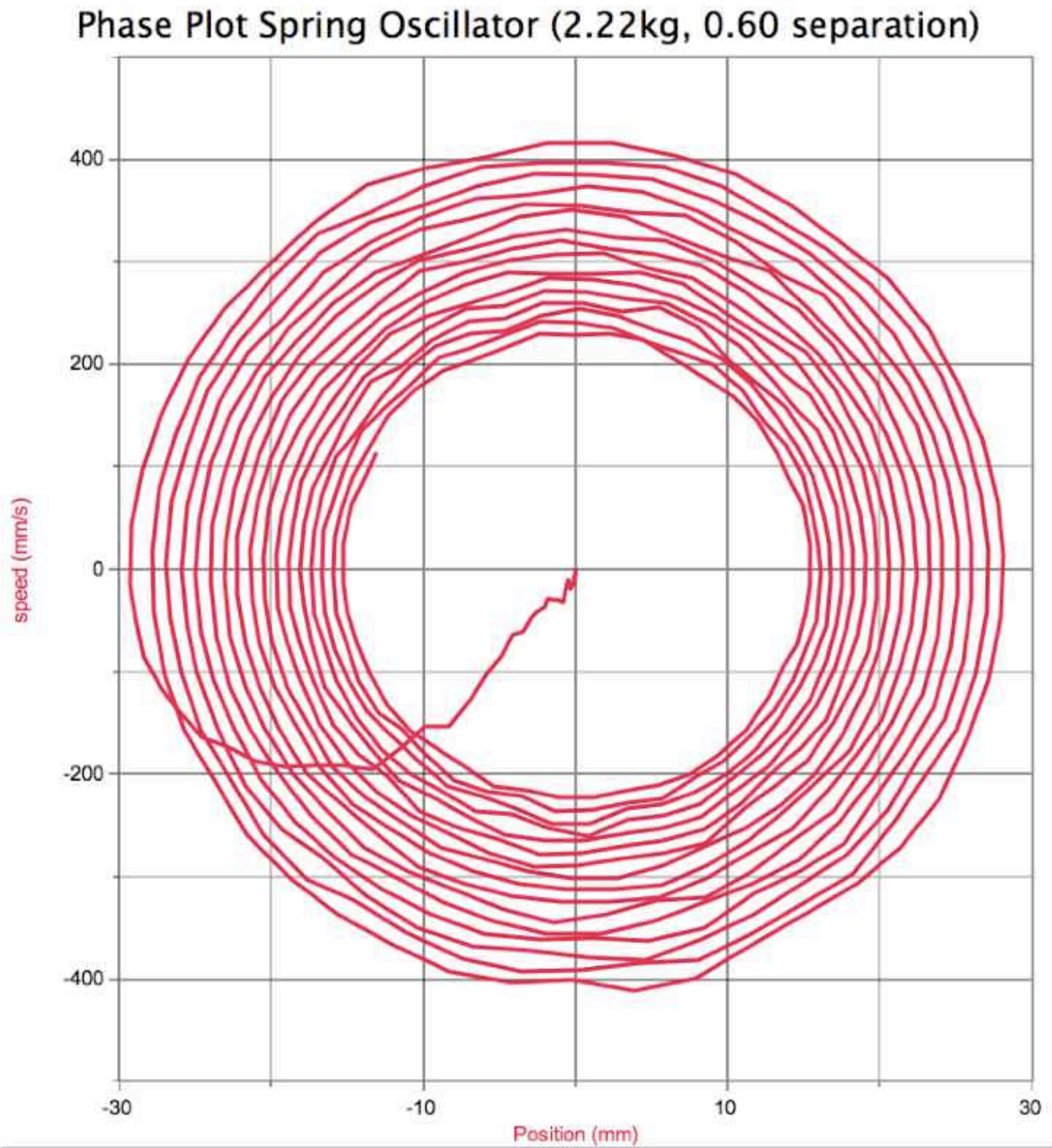


Plate Seven, Spring and Magnetic Restoring Forces



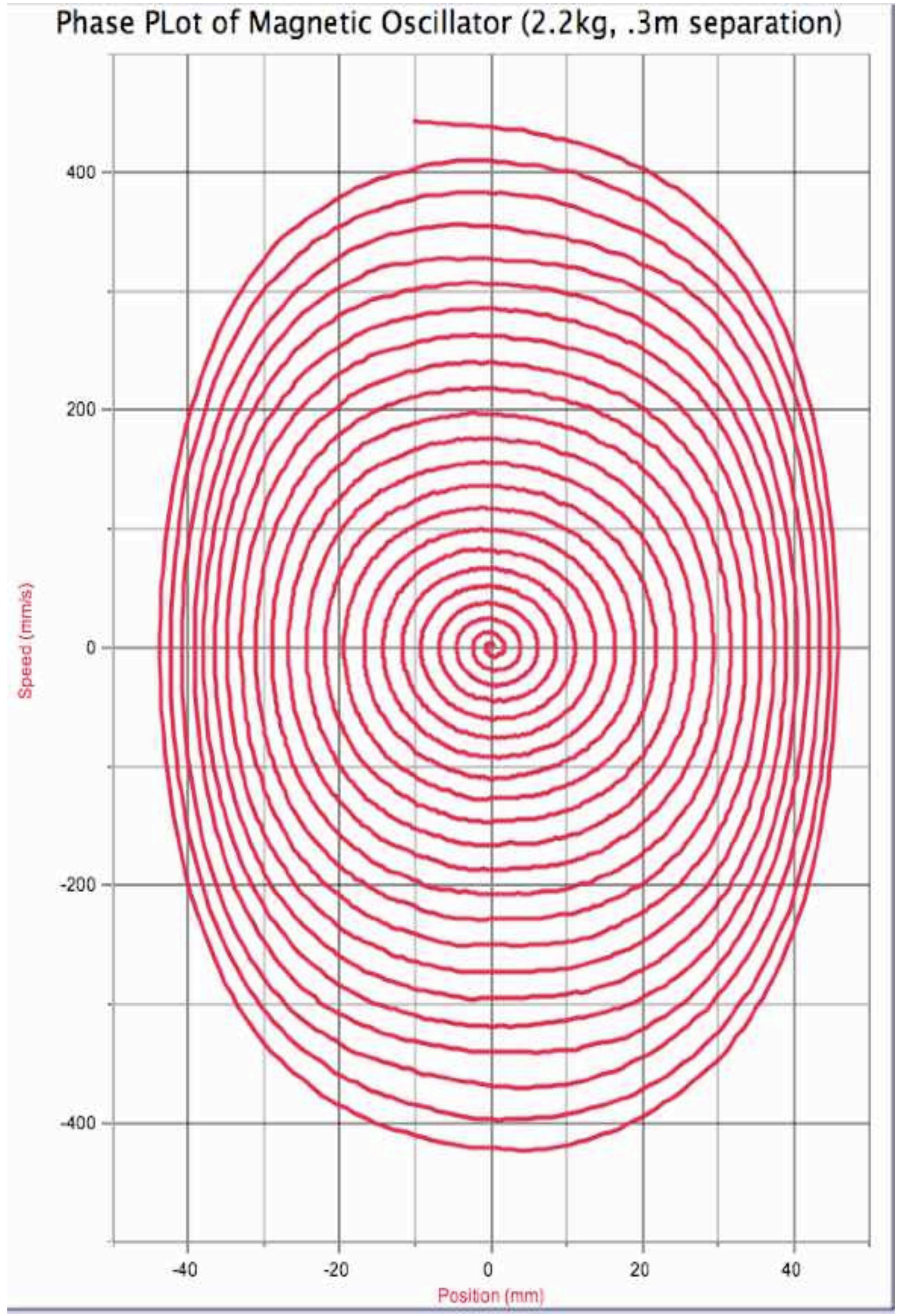


Plate Eight, Phase Plots

```
PRINT  "Hookian Oscillator w/ linear damping"
PRINT  "initial displacement in mm"
PRINT "input displacement in mm"
input x
PRINT x
LET xdot = 0
LET t = 0
PRINT "time increment (delta t)"
!input deltat
 LET deltat = 0.1e-3                ! 0.1 milliS
PRINT deltat
LET m = 2.22
 LET k = 4.86
print "input lin. damping coefficient"
 input dc
!LET dc = 0
PRINT "number of steps (10k steps / sec.) if deltat = .1E-3"
INPUT stop
!LET stop = 30003
PRINT stop
PRINT " T (seconds)       x (mm)"
LET temp = deltat/2               ! calculations in middle of intervals; more accurate

SUB p
    PRINT using "+#.###^^^      +#.###^^^" :T, x
END SUB

SUB step

    LET Q = - x*k/m  - dc * xdot
    LET xdot = Q * temp + xdot      ! = dx / dt
    LET x = x + deltat * xdot
    LET t = t + temp              ! increments time
    LET temp = deltat             !   may be  error should be after sub? after first
increment is delta t, not deltat/2

END SUB

FOR N = 0 to stop

    IF mod (n, 1000) = 0 then CALL p
    !IF n > 9998  and n < 10003 then CALL p

    CALL step

NEXT N
END
```

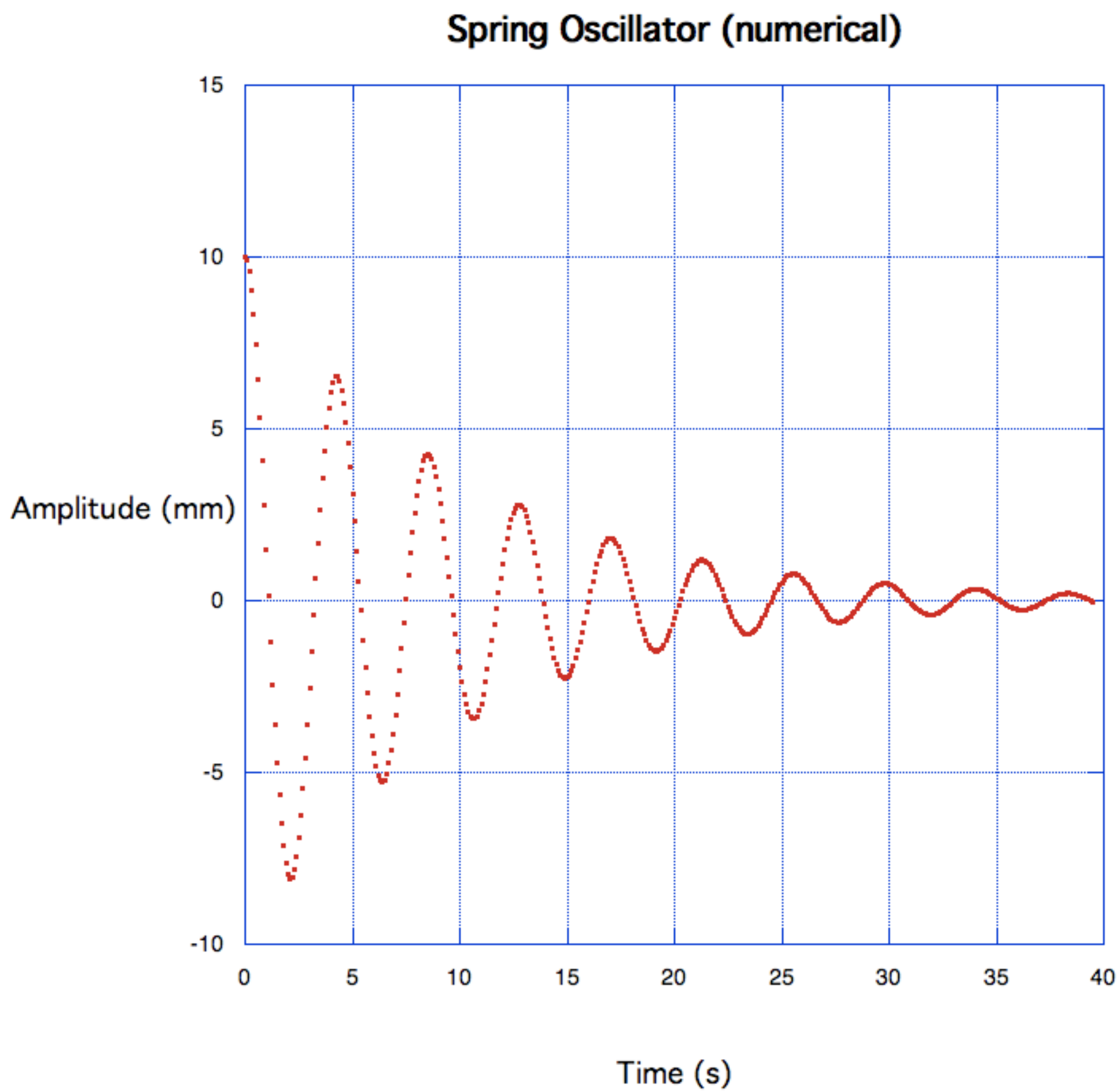


Plate Nine, Leapfrog Algorithm and Example Plot