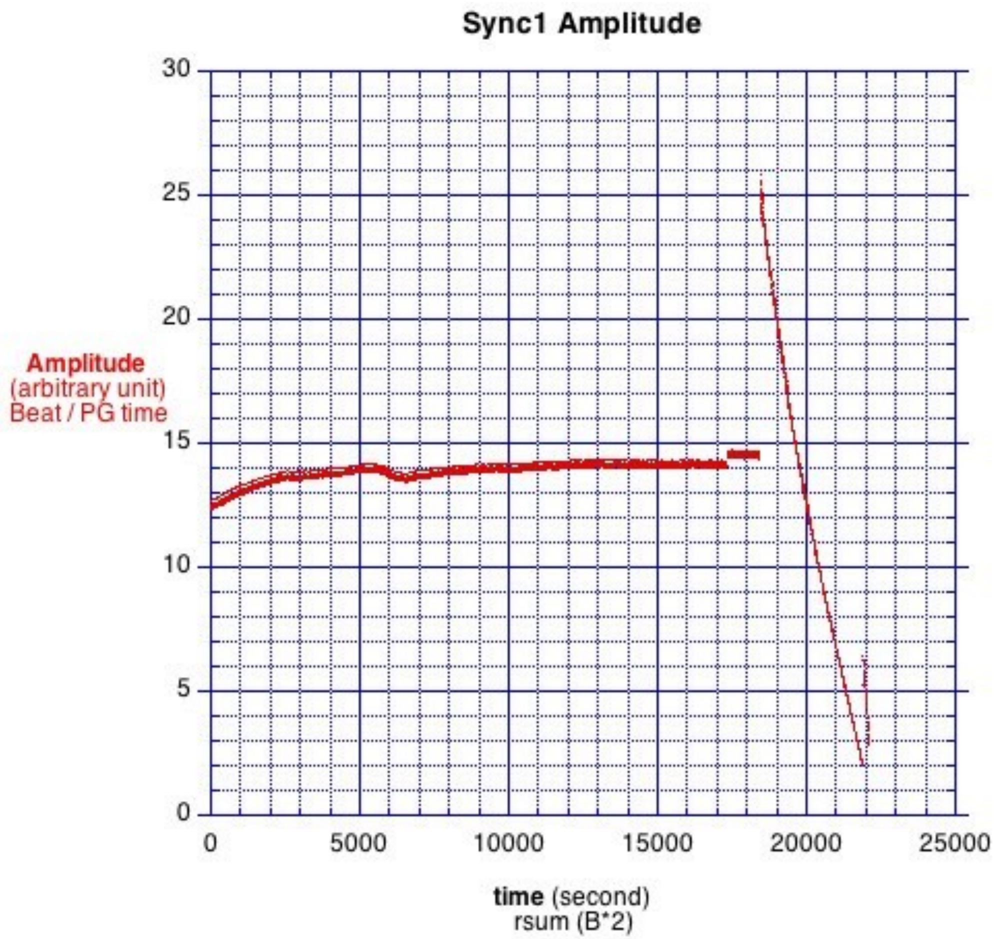
DRAFT-1 Further Analysis of Mr. Mumford's Synchronome, and the Harmonicity of the SWC

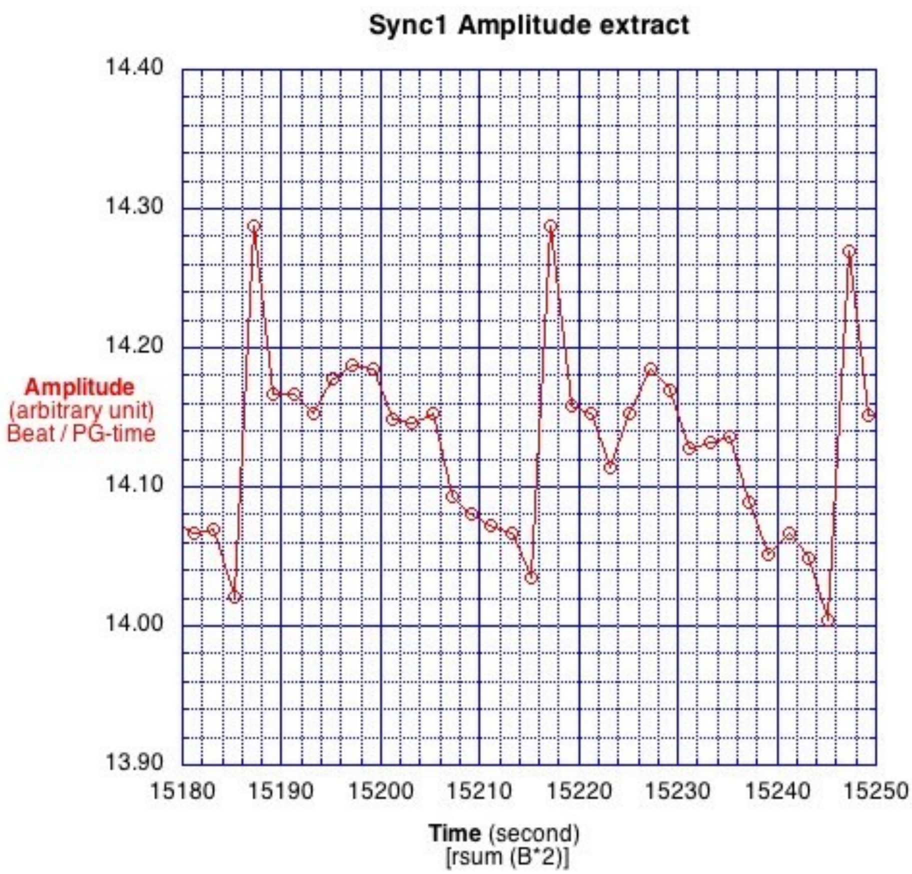
In June 2007 Mr. Mumford sent me data from several free decays of his clock. I didn't realize until a few weeks ago that the first one included the clock running before he disabled the escapement, displaced the pendulum to about double the running amplitude and allowed it to freely decay. [1] I have now used both the running portion and that decay to find the Q while running and at the running amplitude while freely decaying. The graph below shows the amplitude of that first trial (all data).



---

[1] I didn't analyze it, then, because I mistook the running behavior as noise and instead requested further trials until one was satisfactory. That one is my labeled number six, which I used to show my quasi-continuous method for measuring Q [HSN 2007-4] Not: At the time I didn't realize that the decrease in measured Q at small amplitude is due to the finite width of the flag, and not that a constant dissipation dominates, which I had claimed. I am indebted to Mr. D. Drumheller for informing me. [private comm. and HSN 212-4]

Instead of using the time stamp, the time is the sum of the periods.
Since the pendulum is periodically free[2] for about 29 seconds, one may find the free Q during that period. I selected a time when the average amplitude was relatively constant to measure the Q over two free times. A graph of the two I used follows:



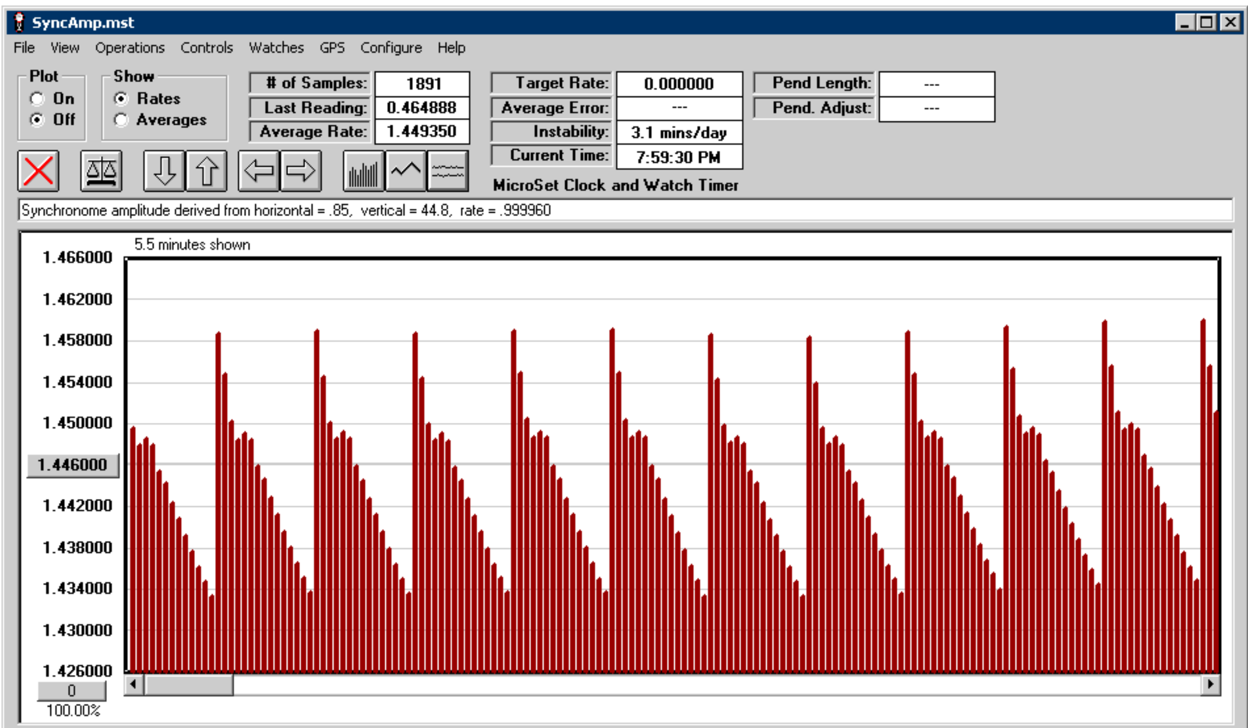
The Q values I found from the two, using their extrema, are 2.5k. However, the pendulum may not be free doing this time, therefore, I also calculated the Q from the

---

2 The header of the second set he sent me: "… 6/8/2007: Synchronome run down, count wire tied up. Started from higher amplitude" I presume this is the method used in all the free decays. I think in all cases the case's door was ajar for the MicroSet's cables, which likely added to the noise.

times 15203 to 15213. Wherein the Q is 2.8k. This agrees rather well with the following free decay shown in the graph above. Note: I have subtracted approximately 18500 seconds from the first graph so this extract begins at zero. The following graph shows a noisy period when an expanded scale is used:

Here below is p. 19 from Bryan's web site: http://www.bmumford.com/mset/symposium/page19.html



Note the non-monotonic decrease. In all cases I found similar behaviour, apparently a Synchronome artifact. A video could possibly reveal the cause.

The Q(s) determined from one of the intermittent decays are 2.63k and 2.73k The second from the part of the decay after the momentary increase in amplitude.

Since it's generally agreed Synchronome Q is largely determined by atmospheric drag, one may use the drag formula[3] to determine the loss. Since the Reynolds number

---

[3] Force = C_D * speed^2 * cross sectional area * air density / 2
https://en.wikipedia.org/wiki/Drag_equation

varies considerably during the period [4], one may, from several published articles[5] find the relationship of the drag coefficient (C_D) with the Reynolds number.
Below ~ **R**e = 300, the C_D is inversely proportional to **R**e. Above 300 (to ~ 50,000) the number is approximately constant at 0.7; therefore, C_D ~= (15/**R**e)+0.7.
Substituting, the force is then: Force = half * air density * speed^2 * cross-sectional area (i.e. length * diameter) of the bob * [{15 * viscosity)/(density* speed * diameter) } + 0.7]

The loss over one period for a damped oscillator is: E = $4\int_0^{T/4} C_D A^3 \omega^3 \sin^3(\omega t)dt$

Substituting: A (amplitude) ~= 30.3 m rad; atmospheric density ~ 1.25 kg/m^3; Area of the bob: L ~= 13.3 e-2 m; D ~= 7.6e-2 m. One obtains ~ 3.25e-6 J for the first term (F ~ V^2), and ~ 2 e-7 for the second (F~ V), respectively greater than and less than Re = 300.

From the definition of Q [E/ΔE], assuming all the loss is atmospheric, Q ~= 8k

I have neglected loss from the rod, which may be significant.[6] Furthermore, my calculation assumes non-turbulent flow upstream. I'm researching the effect of an objected moving in turbulent fluid and hope to report on this later.

It is interesting to compare the Synchronome's loss with that of the Burgess B's loss. Because the bob is nearly a flat plate its loss is principally due to skin friction instead of form or pressure drag. As a first approximation, one may treat the bob as a flat plate. Doing so the Reynolds number's characteristic length is 100 mm and the free stream speed is 0.313 m/s (max.). First, assuming the bob is also square the number is ~ 2.14k. An approximate correction is to reduce this by the ratio of the areas (Pi/4). Re then is ~ = 1.68k. Continuing with the flat plate model: The drag is [1.33 * (kinematic viscosity)^0.5] * air density * (free stream speed) * width * diameter of the bob. Substituting: viscosity = 1.53e-5, and density 1.25 Kg/m^3 one obtains: 2.04e-4 N.

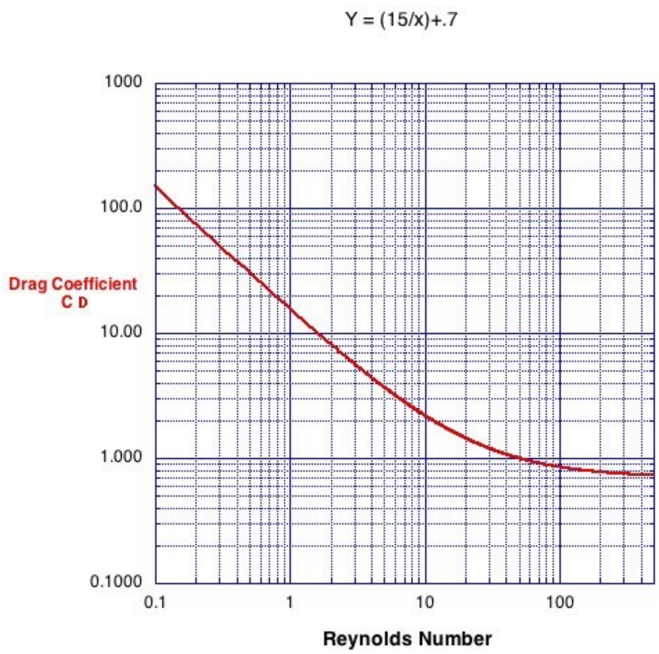
For missing steps, etc. enquire: bernardcleyet@redshift.com

---

[4] From ~ 500 to, of course, zero. https://en.wikipedia.org/wiki/Reynolds_number

[5] http://poisson.me.dal.ca/site2/courses/mech3300/4_Drag.pdf ; http://scienceworld.wolfram.com/physics/CylinderDrag.html

[6] Rawlings (PW) does include rod dissipation p. 102 My treatment is an expansion of Rawlings, pp. 99 ff. My initial very approximate calculation for the rod results in an energy loss of ~14% of the bob's.

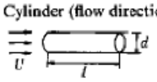
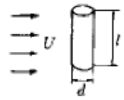
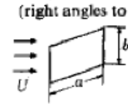
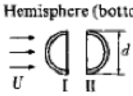
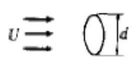


Addendum:
Cd = (15/Re) from interpolation L/D cylinder 2 dimensions and 0.7 from L/D of the bob.
[300< Re < 1E5] [7]

https://www.youtube.com/watch?v=D9UT6YwzmBU Ken Kuo's animation

7 http://www.mech.pk.edu.pl/~m52/pdf/fm/R_09.pdf

Table 9.2 Drag coefficients for various bodies

Body	Dimensional ratio	Datum area, A	Drag coefficient, C_D
Cylinder (flow direction) 	$l/d = 1$		0.91
	2		0.85
	4	$\frac{\pi}{4}d^2$	0.87
	7		0.99
Cylinder (right angles to flow) 	$l/d = 1$		0.63
	2		0.68
	5		0.74
	10	dl	0.82
	40		0.98
	∞		1.20
Oblong board (right angles to flow) 	$a/b = 1$		1.12
	2		1.15
	4		1.19
	10	ab	1.29
	18		1.40
	∞		2.01
Hemisphere (bottomless) 	I	$\frac{\pi}{4}d^2$	0.34
	II		1.33
Cone 	$\alpha = 60°$	$\frac{\pi}{4}d^2$	0.51
	$\alpha = 30°$		0.34
		$\frac{\pi}{4}d^2$	1.2
Ordinary passenger car 		Front projection area 	0.28–0.37

One of the approximations I have used is that the Synchronome pendulum is a simple one. Using g = 9.82 the rod is 0.995 m long. Neglecting the rod and using the bob's dimensions the length of the rod (pivot to top of bob) is 0.926 m. It's 0.933 m to the CoM. Therefore, the simple pendulum approximation is significantly in error.